

VISCOPLASTIC BEHAVIOR OF PLATES
WITH FINITE DEFLECTIONS

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ABSTRACT

A theoretical investigation is developed herein in order to estimate the permanent transverse deflections of rectangular plates when subjected to large dynamic loads. The influence of finite deflections or geometry changes and of strain-rate sensitivity is retained in the analysis but elastic effects are disregarded. The particular case of a fully clamped plate acted on by a uniformly distributed pressure pulse is studied in detail. It is observed that good agreement between the theoretical predictions and experimental results has been obtained for rectangular plates made from strain-rate sensitive material.

Thesis Supervisor

Professor Norman Jones

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NOTATION

B	Semi-width of plate
D	Material constant defined by equation (2)
H	Thickness of plate
I	$p_0 \tau$
I'	$p_0 \tau / (\mu H p_c)^{1/2}$
L	Semi-length of plate
M ₀	Static limiting plastic moment = $\frac{\sigma_0 H^2}{4}$
M' ₀	Dynamic limiting plastic moment, defined by equations (4a-d)
N ₀	Static limiting plastic axial force = $\sigma_0 H$
N' ₀	Dynamic limiting plastic axial force, defined by equations (4a-d)
T, T ₁	Durations of parts of the response
V ₀	Initial velocity
W _m	Permanent transverse deflection
W _f	Permanent transverse deflection if $\gg H$
p	Material constant defined by equation (2)
p _c	Magnitude of static collapse pressure
p _i , p ₃	External pressures per unit surface area of a plate
p ₀	Magnitude of dynamic pressure pulse
t, t ₂ , t ₃	Time
w	Transverse deflection of the midplane of a plate
x, x', y	Co-ordinates defined in figure 2
z	Co-ordinate which is perpendicular to the mid-plane of an initially flat plate
b	$\frac{B}{L}$
ϵ_{ij}	Direct strain
η	$\frac{P_0}{P_c}$

$\dot{\theta}_m$	Relative angular rotation rate across a line hinge
K_{ij}	Curvature
	Mass per unit area of plate
ξ_0	$\tan \phi$
	Non-dimensional times
σ_0	Yield stress
τ	Duration of a rectangular pressure pulse
	angle defined in figure 2
(\cdot)	$\frac{\partial}{\partial t}$
ϵ_0	Strain at middle axis of plate's section

INTRODUCTION

In reference [1] Jones investigated the influence of finite deflections on the behavior of arbitrarily shaped plates and beams which are subjected to large dynamic loads. In his analysis Jones disregarded any elastic effects, strain-hardening and rate sensitivity. It has been shown that elastic effects can be neglected in the computation of final deformation if the kinetic energy input is appreciably larger than the maximum elastic strain energy the plate can absorb. With respect to material properties, experiments have indicated that strain rate sensitivity is by far the most important property neglected in rigid-perfectly plastic methods. Thus, while experiments with rate insensitive materials, such as Aluminium Al6061T6, show remarkable agreement with theoretical results, in the case of mild steel, a notoriously rate sensitive material, significant disagreement between theory and experiment takes place, the net result being an underestimation on the ability of a plate to sustain a given dynamic load. Cowper and Symonds [3,8] and Symonds and Jones [4] have developed an approximate theoretical method to examine the influence of rate sensitivity in beams. Their results show drastic improvement, as compared with experimental evidence, over the non-rate sensitive methods. [2, 5]

A general approximate theoretical procedure, which retains both finite deflections and strain rate sensitivity, is presented herein for the dynamic behavior of arbitrarily shaped, rigid-perfectly plastic plates. This procedure is essentially an extension of the one developed by Jones [1] in order to include strain rate effects. The particular case of a fully clamped rectangular plate subjected to a uniformly distributed dynamic pressure pulse is studied in more detail.

APPROXIMATE ANALYSIS

In reference [1] Jones shows that if a plate (or beam) is divided into a number of rigid regions separated by $\sqrt{}$ straight line hinges, each of length l_m , then an energy balance yields the following relation:

$$\int_A (p_3 - u\dot{w}) \dot{w} dA = \sum_{m=1}^{\sqrt{}} \int_{l_m} (Nw - M) \dot{\theta}_m dl_m \quad (1)$$

where w is the transverse deflection at the hinge.

The stresses at the hinges due to the axial force N and bending moment M act on a plane which is parallel to the hinge and transverse to the midplane of the plate. $\dot{\theta}_m$ is the relative angular rotation rate across the hinge and is supposed to be infinitesimal. The quantity $(Nw - M) \dot{\theta}_m$, on the right hand side of equation (1), represents the internal energy dissipation per unit length of a hinge. This dissipation function will depend on the boundary conditions of a plate and on the yield condition used.

For a strain rate sensitive material Symonds and Jones in reference [3] represent the dependence of the dynamic lower yield stress $\sigma(\dot{\epsilon})$ on the plastic strain rate $\dot{\epsilon}$ by the following:

$$\frac{\sigma(\dot{\epsilon})}{\sigma_0} = 1 + \left(\frac{\dot{\epsilon}}{D} \right)^{\frac{1}{p}} \quad (2)$$

where σ_0 is the static yield stress, and p and D are material constants. This relation is totally empirical and approximate, providing good representation of test results up to strain rates in the neighborhood of $1,000 \text{ sec}^{-1}$. If formula (2) is used to derive dynamic plastic bending moment and axial force for rate sensitive behavior, assuming the Navier hypothesis valid, Symonds and Jones [4] show that:

(see next page)

$$\frac{M}{M_{O'}} = 1 - \frac{4 z^2}{H^2} \quad (4a)$$

$$\frac{N}{N_{O'}} = 2 \frac{z}{H} \quad (4b)$$

where $M_{O'} = \left[1 + \frac{2p}{2p+1} \left(\frac{\dot{K} H}{2D} \right)^{1/p} \right] M_o$

$$N_{O'} = \left[1 + \left(\frac{\dot{K} H}{2D} \right)^{1/p} \right] N_o$$

and $z \leq \frac{H}{2}$

$$\frac{M}{M_{O'}} \approx \frac{1}{2p+1} \cdot \frac{H}{2z}$$

$$\frac{N}{N_{O'}} = 1$$

where $M_{O'} = \left[1 + \left(\frac{\epsilon_o}{D} \right)^{1/p} \right] M_o$

$$N_{O'} = \left[1 + \left(\frac{\epsilon_o}{D} \right)^{1/p} \right] N_o$$

and $z \geq \frac{H}{2}$

\dot{K} = curvature rate

z = distance between middle and neutral axes of the plate's cross-section.

The moment $M_{O'}$ and axial force $N_{O'}$ are to be interpreted as the dynamic equivalents of the static values M_o , N_o .

When equations (4a) and (4b) are combined they give the yield condition:

$$\frac{|M|}{M_{O'}} = 1 - \left(\frac{N}{N_{O'}} \right)^2 \quad \text{YIELD CONDITION} \quad (5)$$

This condition is the maximum normal stress yield criterion, parabolic in shape, as is seen in figure 1.

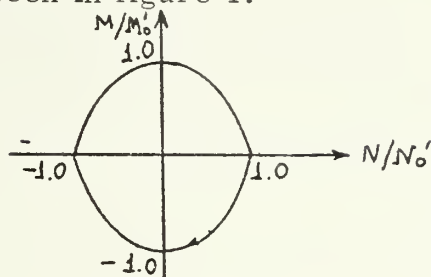


figure 1

It should be pointed out that equation (1) has been derived for arbitrarily shaped plates and since it is an energy balance equation, it can be used with any kind of edge conditions, provided that plastic hinges are allowed to form around the boundary of the plate, if necessary, as well as the interior.

If a kinematically admissible collapse mechanism with straight line hinges is postulated, then equation (1), combined with the appropriate relations for $(Nw - M) \dot{\theta}_m$, can be solved to give an estimate of the influence of finite-deflections and strain rate on the deflection-time history of rigid, perfectly plastic plates (or beams) loaded dynamically.

DYNAMIC BEHAVIOR OF A FULLY CLAMPED RECTANGULAR PLATE

From infinitesimal plasticity analyses it is evident that the shape of the displacement field of a beam or plate for small dynamic loads is the same as the static collapse velocity profile [1,2]. In addition, from experimental results [2,5] it is seen that the permanent deformed profiles of rectangular plates loaded dynamically are similar to the shape of the velocity field used by Wood [6] in static analysis.

Consider a rigid, perfectly plastic rectangular plate of length $2L$ and width $2B$, fully clamped around its outer boundary, as shown in figure 2. The plate is subject to a uniformly distributed dynamic load with the pressure-time history shown in figure 3.

This external loading may be expressed in the form:

$$p_3 = p_0 \quad \text{for } 0 \leq t \leq \tau \quad (6a)$$

$$p_3 = 0 \quad \text{for } \tau \leq t \leq T + \tau \quad (6b)$$

where τ and $(T + \tau)$ are the durations of the pressure pulse and the plate response, respectively.

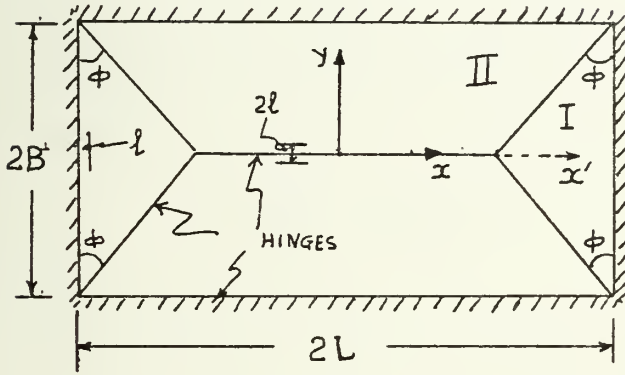


figure 2

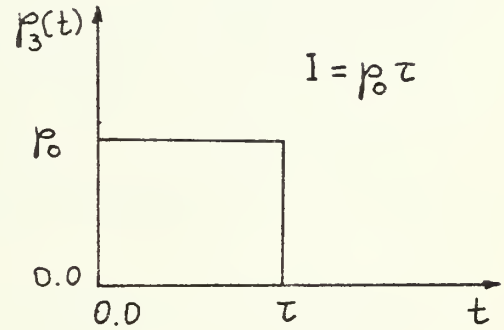


figure 3

Assuming Wood's displacement field, as discussed above, we have:

$$w = \frac{W_i (B \tan \phi - x')}{B \tan \phi} \quad (7a)$$

for region I

and:

$$w = \frac{W_i (B - y)}{B} \quad (7b)$$

for region II. Regions I and II are indicated in figure 2 and $i = 1$ refers to the deflections during the time interval $0 \leq t \leq \tau$, while $i = 2$ refers to $\tau \leq t \leq T + \tau$.

In order to use equation (1), its right hand side is examined first, Thus:

(see next page)

$$\sum_{m=1}^{\sqrt{}} \int_{lm} (Nw - M) \dot{\theta}_m dlm = 4 \int_0^{\frac{B}{\cos \phi}} (Nw - M) \dot{\theta}_m \frac{dx'}{\sin \phi} +$$

$$+ 2 \int_0^{L-B \tan \phi} (Nw - M) \dot{\theta}_m dx + 4 \int_0^L (Nw - M) \dot{\theta}_m dx + 4 \int_0^B (Nw - M) \dot{\theta}_m dy \quad (8)$$

The evaluation of the right hand side of equation (8) requires the use of equations (7a, and 7b) together with the yield condition and the relations:

$$M_o = \frac{\sigma_o H^2}{4}, \quad N_o = \sigma_o H$$

Values for the curvature rate \dot{K} , rotation rate $\dot{\theta}_m$ and hinge width to be used in each integral are given in table (1).

	1 st Integral	2 nd Integral	3 rd Integral	4 th Integral
$\dot{\theta}_m$	$\frac{\dot{W}_1}{B \sin}$	$\frac{2\dot{W}_1}{B}$	$\frac{\dot{W}_1}{B}$	$\frac{\dot{W}_1}{B \tan}$
\dot{K}	$\frac{\dot{W}_1}{2Bl \sin}$	$\frac{\dot{W}_1}{Bl}$	$\frac{\dot{W}_1}{Bl}$	$\frac{\dot{W}_1}{Bl \tan}$
Hinge Width	2l	2l	l	l

Table (1)

It should be pointed out that in the third and fourth integrals $w = 0$.

If the above information together with equations (6a), (7a) and (7b) are substituted into equation (1) then it is straightforward to show that:

(see next page)

$$\ddot{W}_1 + (h_1 W_1^2 + h_2) \dot{W}_1^{1/p} + h_3 W_1^2 = d_1 \quad (9)$$

where: $0 \ll t \ll \tau$, $W \ll H$

$$h_1 = \frac{b_5}{b_1}, \quad h_2 = \frac{b_4}{b_1}, \quad h_3 = \frac{b_2}{b_1}, \quad d_1 = \frac{b_3 - 1}{b_1}$$

$$b_1 = \frac{\mu B^2}{12 Mo} \frac{(2 - b \tan \phi)}{(1 + b \cot \phi)}; \quad b = \frac{B}{L}$$

$$b_2 = \frac{1}{3H^2} \left[1 + 2 \frac{(1 - b \tan \phi)}{(1 + b \cot \phi)} \right]$$

$$b_3 = \frac{Po}{Pc} \equiv \eta, \quad Pc = \frac{12 Mo}{B^2} \frac{(1 + b \cot \phi)}{(3 - b \tan \phi)}$$

$$b_4 = \frac{A_1 b}{(1 + b \cot \phi)}$$

$$b_5 = \frac{A_2 b}{3H^2 (1 + b \cot \phi)}$$

$$A_1 = \frac{C_2 L}{B} - \frac{C_2 \sin^2 \phi + C_1 + C_3 \cos^2 \phi}{2 \sin \phi \cos \phi}$$

$$A_2 = \frac{3C_2 L}{B} - \frac{3C_2 \tan \phi + 3C_1 - C_2 \sin^2 \phi - C_3 \cos^2 \phi}{2 \sin \phi \cos \phi}$$

$$C_2 = \frac{2p}{2p+1} \left(\frac{H}{2 B l D} \right)^{1/p}$$

$$C_1 = \frac{2p}{2p+1} \left(\frac{H}{4 B l D \sin \phi} \right)^{1/p}$$

$$C_3 = \frac{2p}{2p+1} \left(\frac{H}{2 B l D \tan \phi} \right)^{1/p}$$

and $\tan \phi = \sqrt{3 + b^2} - b$ to ensure smallest upper bound to the actual static collapse pressure. [5]

Equation (9) obviously is very difficult to solve analytically. Given the fact that the whole procedure herein is an approximate one, and aiming at providing the designers with relatively simple results, a drastic simplification will be made by setting $p = 1$, $D = 500 \text{ sec}^{-1}$, albeit the proper values of p and D for mild steel are 5 and 40 respectively. Thus, the highly non-linear equation (2) is linearized. Even so, however, it is hoped that the whole procedure will give sufficient insight into the behavior of strain rate sensitive plates.

Now equation (9) may be recast as:

$$\ddot{W}_1 + (h_1 W_1^2 + h_2) \dot{W} + h_3 W_1^2 = d_1 \quad (9a)$$

A series solution to equation (9a) employing the method of undetermined coefficients, and which satisfies the initial conditions $W_1 = \dot{W}_1 = 0$ at $t = 0$, yields the results:

$$W_1 = \bar{W}_1$$

and $\dot{W}_1 = \dot{\bar{W}}_1$

at the end of the first stage of motion ($t = \tau$), where:

$$\begin{aligned}\bar{W}_1 = & \frac{d_1 \tau^2}{2} \left[1 - \frac{h_2 \tau}{3} + \frac{h_2^2 \tau^2}{12} - \frac{h_2^3 \tau^3}{60} + \frac{h_2^4 \tau^4}{360} - \right. \\ & - \frac{d_1 h_3 \tau^4}{60} + \frac{h_3 d_1 \tau^5}{420} - \frac{h_2^4 \tau^5}{2520} + \frac{h_2 h_3 d_1 \tau^5}{126} - \\ & \left. - \frac{h_1 d_1^2 \tau^5}{84} \right] \quad (9b)\end{aligned}$$

$$\begin{aligned}\dot{\bar{W}}_1 = & d_1 \tau \left[1 - \frac{h_2 \tau}{2} + \frac{h_2^2 \tau^2}{6} - \frac{h_2^3 \tau^3}{24} + \frac{h_2^4 \tau^4}{120} - \right. \\ & - \frac{h_3 d_1 \tau^4}{20} + \frac{h_3 d_1 \tau^5}{120} - \frac{h_2^4 \tau^5}{720} + \\ & \left. + \frac{h_2 h_3 d_1 \tau^5}{36} - \frac{h_1 d_1^2 \tau^5}{24} \right] \quad (9c)\end{aligned}$$

A study of the second stage of motion ($\tau \leq t \leq T + \tau$)

proceeds in a manner similar to that outlined above for the first stage, but with equations (9b) and (9c) as initial conditions, and with $\eta = 0$.

Thus the differential equation to be solved now is:

$$\ddot{W}_2 + (h_1 W_2^2 + h_2) \dot{W}_2 + h_3 W_2^2 = d_2 \quad (10)$$

where $d_2 = \frac{-1}{b_1}$; $(\dot{}) = \frac{d}{dt_1}$

A series solution to equation (10) employing the method of undetermined coefficients and satisfying equations (9b) and (9c) gives the maximum value of the transverse deflection of the permanently deformed plate as:

$$\frac{W_m}{H} = \frac{W_2(T)}{H} = \frac{\bar{W}_1}{H} + \frac{\dot{\bar{W}}_1}{H} \rho + \frac{\alpha_2 \tau^2 \rho^2}{H} + \frac{\alpha_3 \tau^3 \rho^3}{H} + \frac{\alpha_4 \tau^4 \rho^4}{H}$$

where: $\alpha_2 = \frac{d_2}{2} - \frac{1}{2} (h_2 + h_1 \bar{W}_1^2) \frac{\dot{\bar{W}}_1}{\bar{W}_1} - \frac{1}{2} h_3 \bar{W}_1^2$

$$\alpha_3 = -\frac{1}{3} [\alpha_2 (h_2 + h_1 \bar{W}_1^2) + (h_3 + h_1 \frac{\dot{\bar{W}}_1}{\bar{W}_1}) \bar{W}_1 \dot{\bar{W}}_1]$$

$$\alpha_4 = - [2\alpha_2 \bar{W}_1 \cdot (3h_1 \frac{\dot{\bar{W}}_1}{\bar{W}_1} + h_3) + 3\alpha_3 (h_2 + h_1 \bar{W}_1^2) + (h_3 + h_1 \frac{\dot{\bar{W}}_1}{\bar{W}_1}) \frac{\dot{\bar{W}}_1^2}{\bar{W}_1}]$$

and $\rho = \frac{T}{\tau}$ is the smallest non-negative root of the polynomial equation:

$$4\alpha_4 \tau^3 \rho^3 + 3\alpha_3 \tau^2 \rho^2 + 2\alpha_2 \tau \rho + \dot{\bar{W}}_1 = 0 \quad (12)$$

Equation (12) is obtained from the requirement that $\dot{W}_1 = 0$ at $t_1 = T$ where t_1 refers to second stage alone.

Equation (11) remains valid provided $\frac{W_m}{H} \ll 1$, because when $\frac{W_m}{H} \gg 1$ the dissipation expressions $(Nw - M) \dot{\theta}_m$ take on new values (since then $N = N_0'$ and $M = 0$) on those portions of the hinge lines which have transverse deflections greater than the plate thickness. In this case, a time dependent rectangular boundary travels outwards from the central line hinge toward the edge of the plate. This boundary always has a deflection $W = H$ and divides the plate into two regions:

an inner zone with $W > H$ and an outer zone with $W < H$. The dissipation relations $(Nw - M) \dot{\theta}_m$ must be evaluated in the inner region with $N = N_0'$, $M = 0$ while in the outer region the evaluation which was used earlier remains unchanged. Figure 4 indicates the two regions:

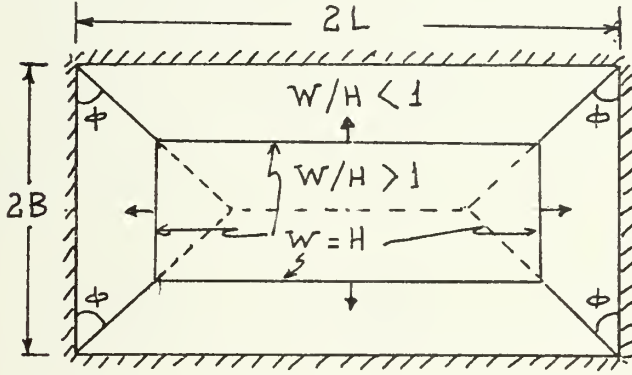


figure 4

If it is assumed that W/H exceeds one when $t \gg \tau$, then the first stage of motion given by equations (9a, b, and c) remains unchanged, while the second stage remains valid until $t = t_2$, when $W/H = 1$ where t_2 is the response time of the second stage alone. Thus, the non-dimensional time $p_2 = \frac{t_2}{\tau}$ may be found from equation (11) with $W = H$, i.e.:

$$\alpha_4 \tau^4 p_2^4 + \alpha_3 \tau^3 p_2^3 + \alpha_2 \tau^2 p_2^2 + \bar{W}_1 \tau p_2 + \bar{W}_1 - H = 0 \quad (13)$$

The smallest non-negative root of (13) gives p_2 , i.e. the response time for the second stage.

Now all subsequent behavior of the plate will have $W \gg H$ at the centrally located plastic hinge. It is therefore necessary to consider a third stage of motion which is governed by the dissipation relations employing $N = N_0'$, $M = 0$ where necessary, as discussed earlier. It is quite straightforward to show that in this case the following equation is obtained:

(see next page)

$$\ddot{W}_3 + Z_1 W_3 + \frac{Z_2}{W_3} + Z_3 W_3 \dot{W}_3^{1/p} + Z_4 \frac{\dot{W}_3^{1/p}}{W_3} = 0 \quad (14)$$

Again the simplification $p = 1$ is employed so that (14) becomes:

$$\ddot{W}_3 W_3 + (Z_3 W_3^2 + Z_4) \dot{W}_3 + Z_1 W_3^2 + Z_2 = 0 \quad (14a)$$

where $t_2 \leq t \leq T + t_2$, $p_3 = 0$

$$Z_1 = \frac{\pi_2}{\pi_1} , \quad Z_2 = \frac{\pi_3}{\pi_1} , \quad Z_3 = \frac{\pi_4}{\pi_1} , \quad Z_4 = \frac{\pi_5}{\pi_1}$$

$$\pi_1 = \frac{\mu(2 - \xi_0)}{Pc(3 - \xi_0)} ; \quad \xi_0 = b \tan \phi$$

$$\pi_2 = \frac{2(3 - 3\xi_0 + \xi_0^2)}{H(3 - \xi_0)}$$

$$\pi_3 = \frac{Hb}{3} \frac{(\cot \phi + \tan \phi)}{(1 + b \cot \phi)}$$

$$\pi_4 = \frac{C_2}{H} \left[\frac{2 - 2\xi_0 + \xi_0 \left(\frac{2 \sin^2 \phi}{\sin^2 \phi} \right)^{1/p}}{(1 + b \cot \phi)} \right]$$

$$\pi_5 = \frac{C_2 b H}{3} \frac{[\cot \phi (\tan \phi)^{-1/p} + \tan \phi]}{(1 + b \cot \phi)}$$

A series solution to equation (14a) with the initial conditions $W_3 = H$,

$\dot{W}_3 = \dot{W}_2$ at $t_3 = 0$, where t_3 is referring to third stage alone, gives:

$$W_3 = H + \dot{W}_2 t_3 + \alpha'_2 t_3^2 + \alpha'_3 t_3^3 \quad (15a)$$

where:

$$\alpha'_2 = -\frac{1}{2H} [Z_1 H^2 + Z_2 + (Z_4 + Z_3 H^2) \dot{\bar{W}}_2]$$

$$\alpha'_3 = -\frac{1}{3H} [Z_1 H \dot{\bar{W}}_2 + Z_3 H \dot{\bar{W}}_2^2 + \alpha'_2 (\dot{\bar{W}}_2 + Z_4 + Z_3 H^2)]$$

and

$$\dot{\bar{W}}_3 = \dot{\bar{W}}_2 + 2\alpha'_2 t_3 + 3\alpha'_3 t_3^2 \quad (15b)$$

when $t_3 = T_1$, $\dot{\bar{W}}_3 = 0$, $W_3 = W_f$

Substituting for T_1 in equation (15b) and retaining its smallest non-negative root, T_1 is obtained, i.e. the response time of the third stage alone. The value of the final and permanent displacement is given then by equation (15a) with $t_3 = T_1$.

$$\frac{W_f}{H} = 1 + \frac{\dot{\bar{W}}_2 T_1}{H} + \frac{\alpha'_2 T_1^2}{H} + \frac{\alpha'_3 T_1^3}{H} \quad (16)$$

where $\dot{\bar{W}}_2$ is obtained from equation (13) by differentiation:

$$\dot{\bar{W}}_2 = \dot{\bar{W}}_1 + 2\alpha_2 \tau p_2 + 3\alpha_3 \tau^2 p_2^2 + 4\alpha_4 \tau^3 p_2^3 \quad (16a)$$

An alternative approximate solution to equation (14a), for the third stage of motion, which is valid for any value of p , but for only a short range of values of the aspect ratio b , can be obtained as follows:

Call $W_4 = \frac{W_3}{H}$

$$Z'_1 = HZ_1 , \quad Z'_3 = HZ_3$$

$$Z'_2 = \frac{Z_2}{H} , \quad Z'_4 = \frac{Z_4}{H}$$

$$\alpha_1 = \frac{Z'_1}{Z'_3}, \quad \alpha_2 = \frac{Z'_2}{Z'_4}$$

Then equation (14a) becomes

$$\ddot{W}_4 + Z'_3 W_4 (\alpha_1 + \dot{W}_4^{1/p}) + \frac{Z'_4}{W_4} (\alpha_2 + \dot{W}_4^{1/p}) = 0 \quad (17)$$

By inspection the constants α_1 and α_2 , which both depend on the aspect ratio b , are approximately equal for $0 \leq b \leq 0.3$, their ratio varying between 0.98 and 1.16.

Thus, by the substitution $\alpha_1 = \alpha_2 = \alpha$, equation (17) is further transformed to:

$$\frac{\ddot{W}_4}{(\alpha + \dot{W}_4^{1/p})} + Z'_3 W_4 + \frac{Z'_4}{W_4} = 0 \quad (17a)$$

Employing the change of variables

$$\dot{W}_4 = \dot{W}_4 \frac{d \dot{W}_4}{d W_4} \quad \text{and integrating:}$$

$$\begin{aligned} & \dot{W}_4^{1-1/p} d\dot{W}_4 - \frac{1}{\alpha} \left[\int \dot{W}_4^{1-2/p} d\dot{W}_4 - \frac{1}{\alpha} \left[\int \dot{W}_4^{1-3/p} d\dot{W}_4 - \right. \right. \\ & \left. \left. - \frac{1}{\alpha} \int \dot{W}_4^{1-4/p} d\dot{W}_4 - \frac{1}{\alpha} \left(\dot{W}_4^{1-5/p} d\dot{W}_4 - \frac{1}{\alpha} \int \frac{\dot{W}_4^{1-5/p}}{\alpha + \dot{W}_4^{1/p}} d\dot{W}_4 \right) \right] + \right. \\ & \left. + \frac{Z'_3}{2} W_4^2 + Z'_4 \log_e W_4 + C = 0 \right] \quad (17b) \end{aligned}$$

For mild steel, when $p = 5$, equation (17b) yields:

(see next page)

$$\begin{aligned}
& \frac{1}{2} \alpha^{-7} \dot{W}_4^2 + \left(\frac{5}{4} - \frac{5}{9} \alpha^{-\frac{34}{5}} \right) \dot{W}_4^{9/5} + \left(\frac{5}{8} \alpha^{-\frac{33}{5}} - \frac{5}{3} \alpha^{-1} \right) \dot{W}_4^{8/5} + \\
& + \left(\frac{5}{2} \alpha^{-2} - \frac{5}{7} \alpha^{-\frac{32}{5}} \right) \dot{W}_4^{7/5} + \left(\frac{5}{6} \alpha^{-\frac{31}{5}} - 5 \alpha^{-3} \right) \dot{W}_4^{6/5} + \\
& + (\alpha^{-4} - \alpha^{-6}) \dot{W}_4 + \frac{Z'_3}{2} W_4^2 + Z'_4 \log_e W_4 + C = 0
\end{aligned}$$

The constant C is evaluated with the conditions:

$$\text{at } t = t_2 \quad W_4 = \frac{W_3}{H} = 1, \quad \dot{W}_4 = \frac{\dot{W}_2}{H}$$

Then the conditions

$$\text{at } t = T_1 \quad \dot{W}_4 = 0, \quad W_4 = \frac{W_f}{H} \quad \text{give:}$$

$$\begin{aligned}
& \frac{Z'_3}{2} \left(\frac{W_f}{H} \right)^2 + Z'_4 \log_e \left(\frac{W_f}{H} \right) = \frac{1}{2} \alpha^{-7} \left(\frac{\dot{W}_2}{H} \right)^2 + \\
& + \left(\frac{5}{4} - \frac{5}{9} \alpha^{-\frac{34}{5}} \right) \left(\frac{\dot{W}_2}{H} \right)^{9/5} + \left(\frac{5}{8} \alpha^{-\frac{33}{5}} - \frac{5}{3} \alpha^{-1} \right) \left(\frac{\dot{W}_2}{H} \right)^{8/5} + \\
& + \left(\frac{5}{2} \alpha^{-2} - \frac{5}{7} \alpha^{-\frac{32}{5}} \right) \left(\frac{\dot{W}_2}{H} \right)^{7/5} + \left(\frac{5}{6} \alpha^{-\frac{31}{5}} - 5 \alpha^{-3} \right) \left(\frac{\dot{W}_2}{H} \right)^{6/5} + \\
& + (\alpha^{-4} - \alpha^{-6}) \left(\frac{\dot{W}_2}{H} \right) + \frac{Z'_3}{2} \tag{18}
\end{aligned}$$

The final permanent deflection is given by the root of transcendental equation (18) and can be expected to be within the interval:

(see next page)

$$\begin{aligned}
1 \ll \frac{W_f}{H} \ll & \left[1 + \frac{2}{Z'_3} \left[\frac{1}{2} \alpha^{-7} \left(\frac{\dot{W}_2}{H} \right) + \left(\frac{5}{4} - \frac{5}{9} \alpha^{-\frac{34}{5}} \right) \left(\frac{\dot{W}_2}{H} \right)^{9/5} \right. \right. \\
& + \left(\frac{5}{8} \alpha^{-\frac{33}{5}} - \frac{5}{3} \alpha^{-1} \right) \left(\frac{\dot{W}_2}{H} \right)^{8/5} + \left(\frac{5}{2} \alpha^{-2} - \frac{5}{7} \alpha^{-\frac{32}{5}} \right) \left(\frac{\dot{W}_2}{H} \right)^{7/5} + \\
& \left. \left. + \left(\frac{5}{6} \alpha^{-\frac{31}{5}} - 5 \alpha^{-3} \right) \left(\frac{\dot{W}_2}{H} \right)^{6/5} + \left(\alpha^{-4} - \alpha^{-6} \right) \left(\frac{W_2}{H} \right) \right] \right]^{1/2}
\end{aligned}$$

DISCUSSION

The approximate method presented in this paper has been used to predict the permanent transverse deflections (W_f/H) presented in Figures (5-9) for fully clamped, rate-sensitive rectangular plates which are subjected to uniformly distributed impulsive loading, i.e. when $p_0 \tau = \mu V_0$.

It is evident from figure (5) and tables (2-6, 9) that the estimates which were obtained using equations (11) or (16) and rectangular plates with $b = 0.593$ agree reasonably well with the corresponding experimental values.

It is apparent from figures (6- 8) that, for a given value of impulse, the permanent transverse deflection of a plate is essentially independent of the magnitude of η when η is larger than 50. It is also noticed that for increasing values of η the deflections slightly decrease, whereas in the non-rate-sensitive analysis of Ref. [1] they are seen to slightly increase. The probable explanation for this difference in behavior could be that in the rate-sensitive case with increasing η the rate-sensitive effects become more pronounced the net result being the (favorable) reduction in W_f/H . However, since for large values of η the deflections are essentially independent of η , as discussed above, for impulsive loading, when $\eta \rightarrow \infty$, this difference in behavior becomes insignificant.

From figures (9, 10) and tables (9-11) it is observed that the value of the permanent deflection is also affected by the value of the hinge width and the value of the material property D . It should be pointed out here that the value of D is merely a value used in equation (2) to describe the $\sigma - \dot{\epsilon}$ curve for the material. The proper value of D for mild steel is 40 sec^{-1} when $p = 5$. The mathematical difficulties that led to the linearization of equation (2) by setting $p = 1$ naturally have an effect on the value of D as well, thus affecting the final results. However,

it is evident from figure () that for values of D greater than 300 sec^{-1} the variation in W_f/H is rather small so that if $D = 300 \text{ sec}^{-1}$ is used with the present analysis results good enough for engineering predictions can be obtained. Similarly, the optimum value of the hinge width, which is unknown at the outset, may be seen from figure () to be $l = 5H$. (These observations are made for the particular case of mild steel).

All the theoretical results presented in this paper were obtained by assuming that angular changes across the plastic hinges were sufficiently small to permit $\tan \phi$ to be replaced by θ rad. This simplification provides a good approximation when W_f/H is not too large. Moreover, only the few first terms were retained in the series solutions of the differential equations encountered in the analysis. It would be necessary to consider additional terms for large W_f/H ratios and for cases in which the dynamic to static pressure ratio (η) is small. The yield condition derived in ref [4] and presented here as equation (2) is an approximation to the exact one, though the effect of this approximation is not considered to be important. The linearization of equation (2) by setting $p = 1$ is considered to be of some importance since then a highly non-linear curve is replaced by a straight line. However, given the fact that the most important factor in dynamic loading of plates is the influence of finite deflections, which are ignored in bending-only methods, this linearization seems to be acceptable, at least tentatively, since the results are reasonably close to experimental values and at the same time a good insight into the rate-sensitive behavior is gained.

The approximate procedure presented in this paper could be used to examine the influence of large transverse dynamic loads on the behavior of beams and plates which have any shape and support conditions, provided the appropriate plastic hinge pattern is postulated.

Finally it should be remarked that the predictions of this analysis are thought to be valid when the external dynamic energy applied to a

beam or plate is appreciably larger than the amount of energy which could be absorbed in an elastic manner.

CONCLUSIONS - RECOMMENDATIONS

An approximate theoretical investigation is developed herein in order to estimate the permanent deflections of rectangular plates and beams subjected to large dynamic loads. The influence of geometry changes and of strain-rate sensitivity is retained in the analysis but no elastic effects are considered. The particular case of a fully clamped rectangular plate acted on by a uniformly distributed pressure pulse is studied in detail. Tables and graphs are provided for the case of fully clamped plates and beams made from mild steel and loaded impulsively. It is observed that good agreement between the theoretical predictions and experimental results has been obtained for a rectangular plate of aspect ratio $b = 0.593$. Improved theoretical predictions are expected to be obtained if equation (11) is solved for the correct value of p . Though equation (18) is an exact solution to equation (17), it is valid only for a short range of aspect ratios ($0 - 0.3$) and even in that range it contains some approximations. Moreover, equation (17) corresponds to the third stage of motion only so that the exact solution (18) should not be used unless equation (11) has been solved correctly as well. Exact or numerical solutions to equations (11) and (16) would be a useful complement to this paper.

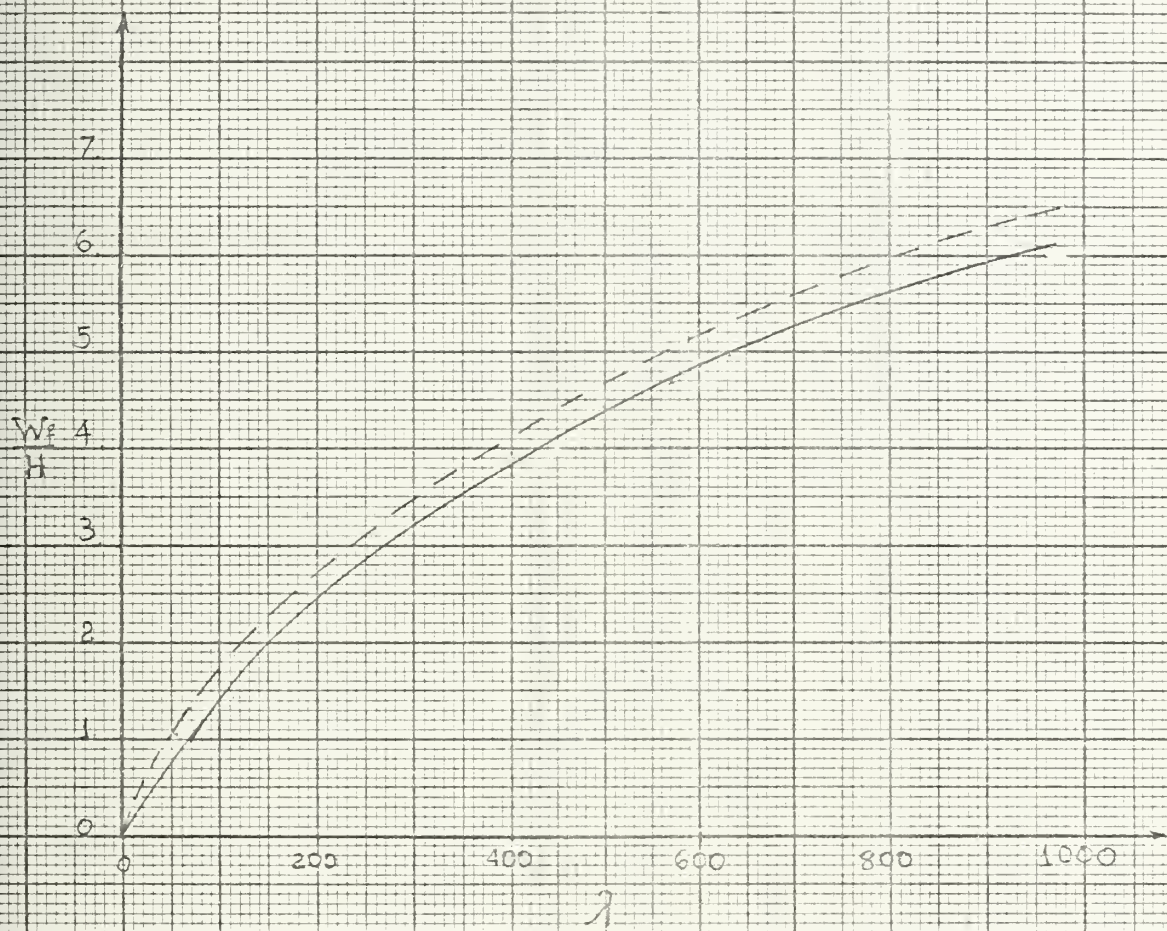


Figure (5) ——— Equation (11) or (16)
 ——— Experimental, Ref [2]
 - · - · - $b = .523$

ASPECT RATIO=.593

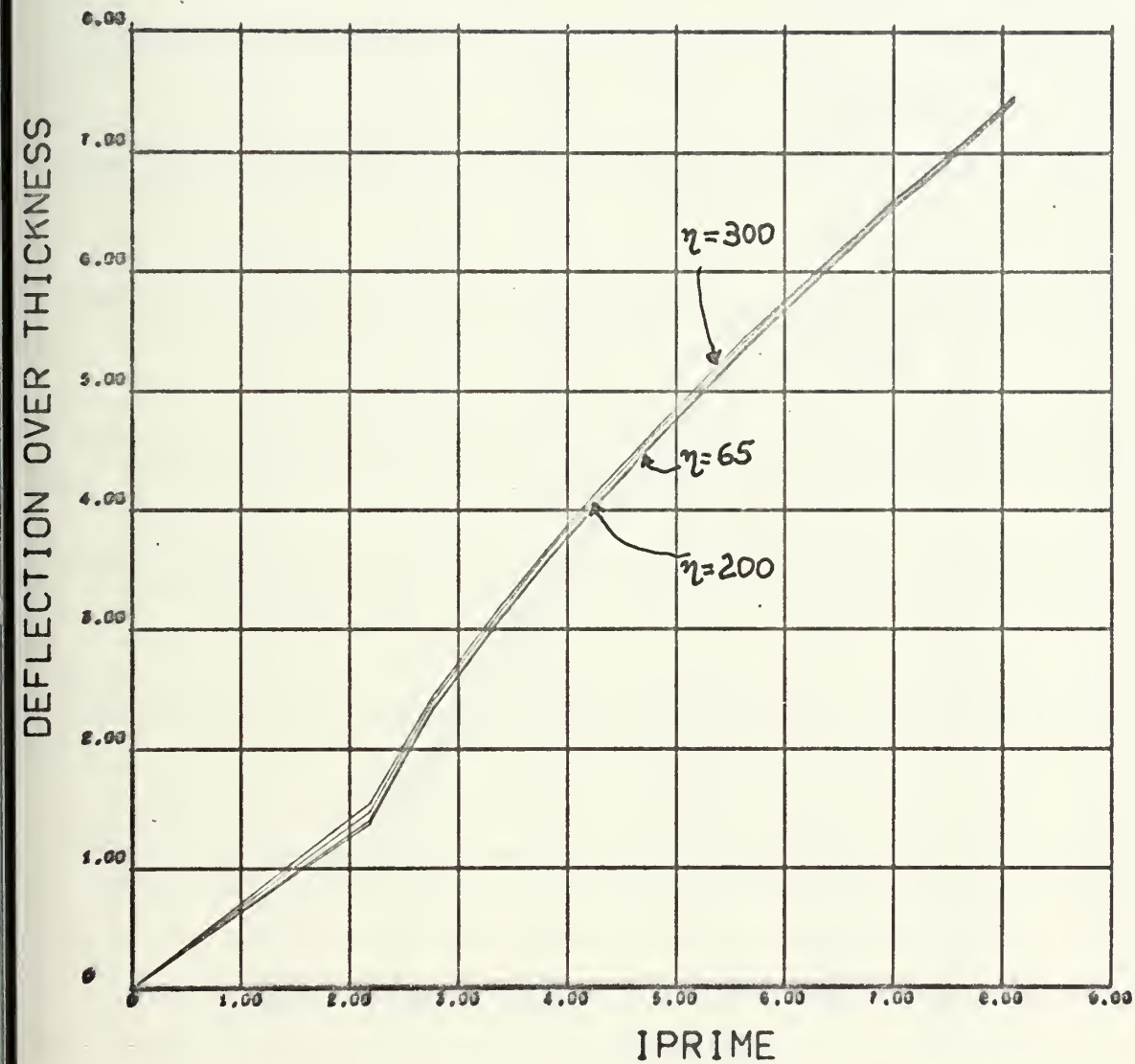


FIG. (6)

ASPECT RATIO=.75

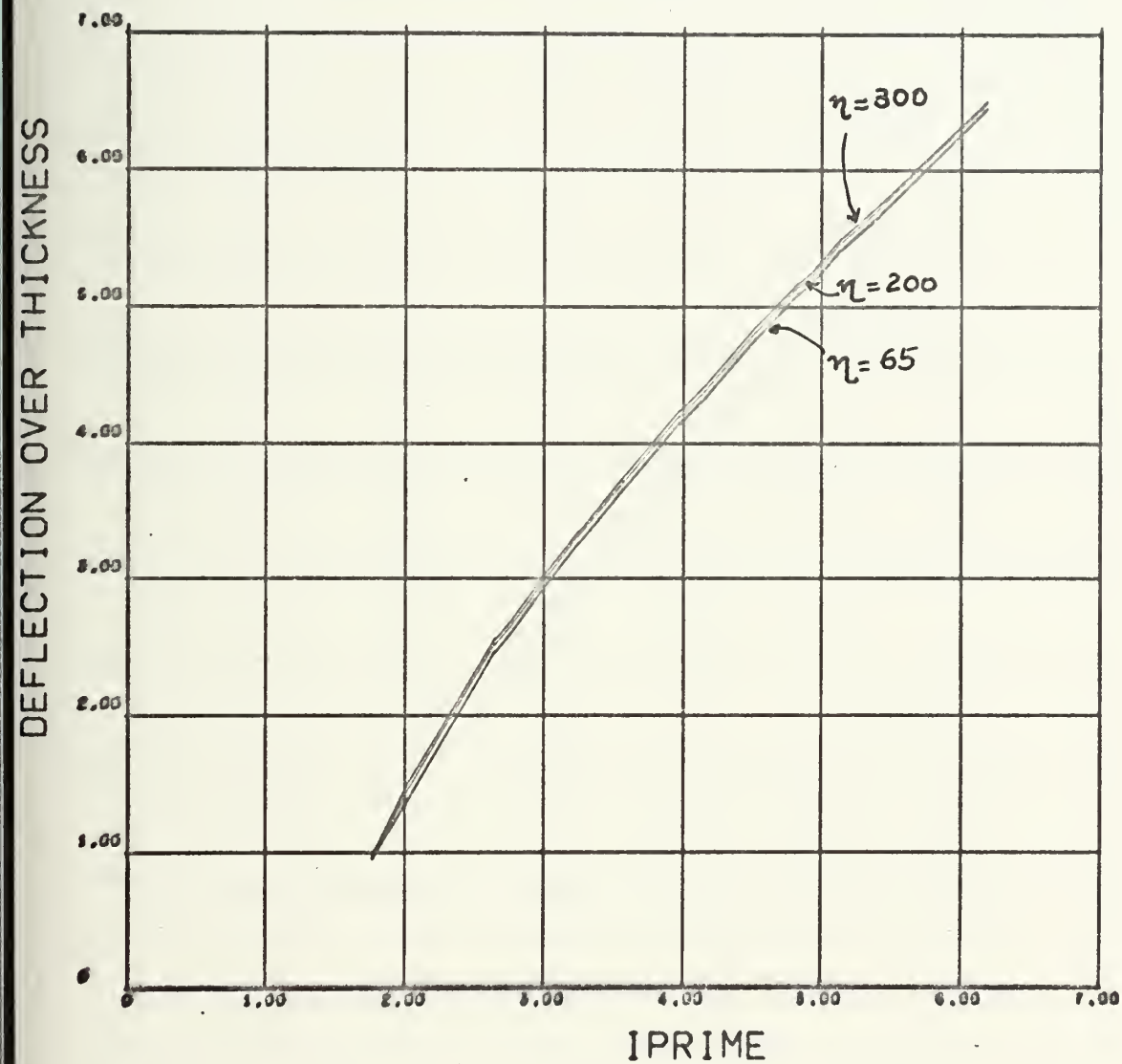


FIG. (7)

ASPECT RATIO=1.0

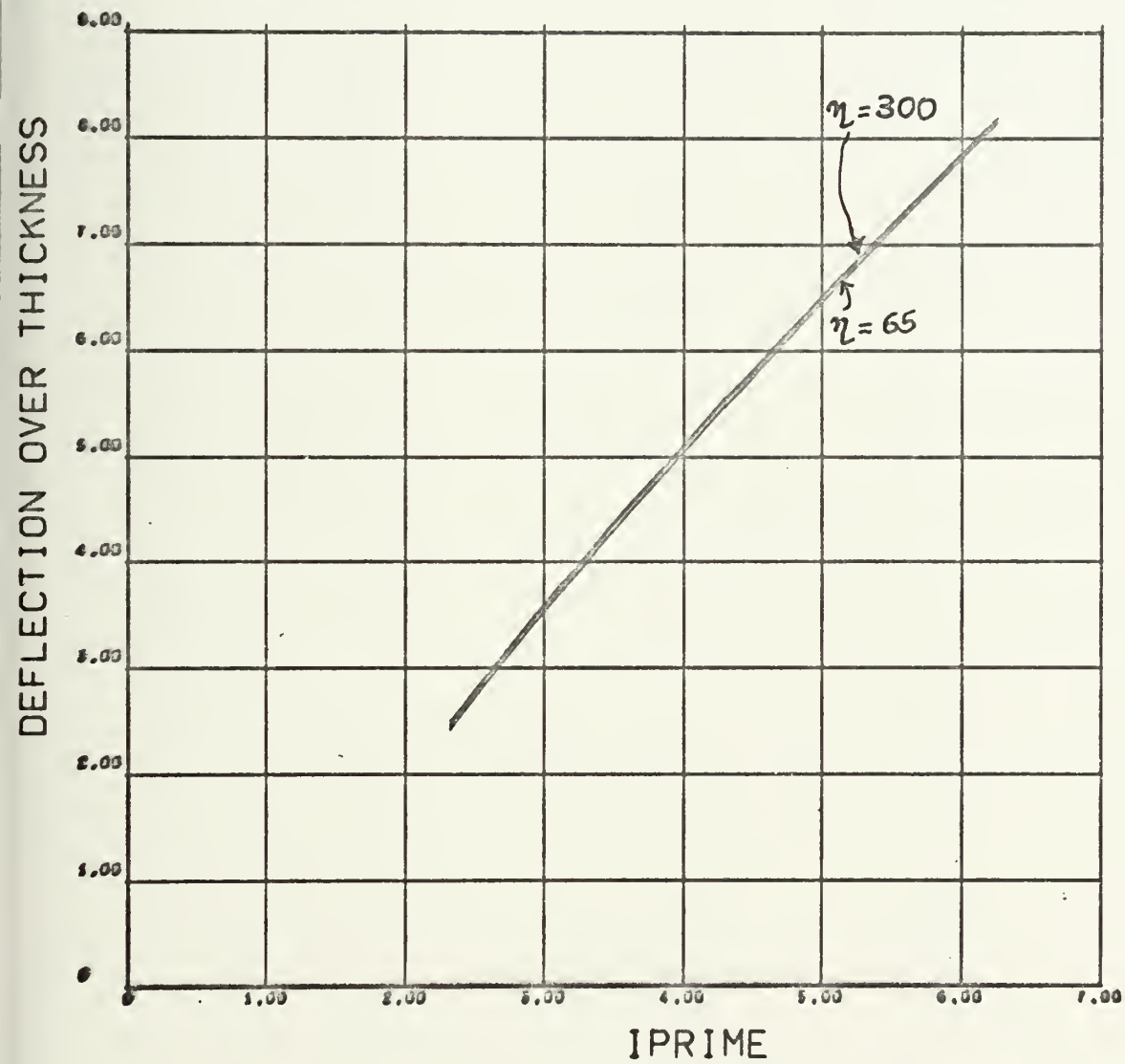


FIG. (8)

W_i
H

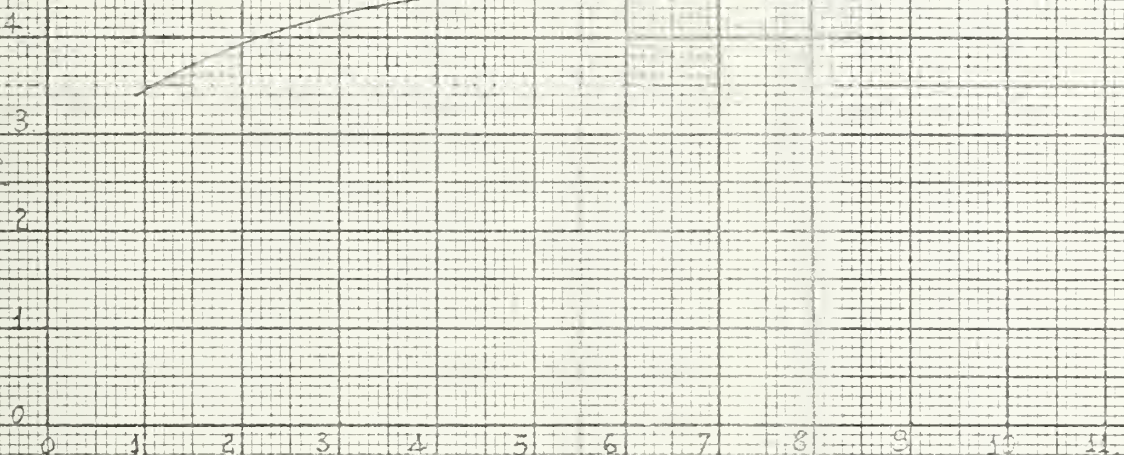


Figure (9)

W_F
H

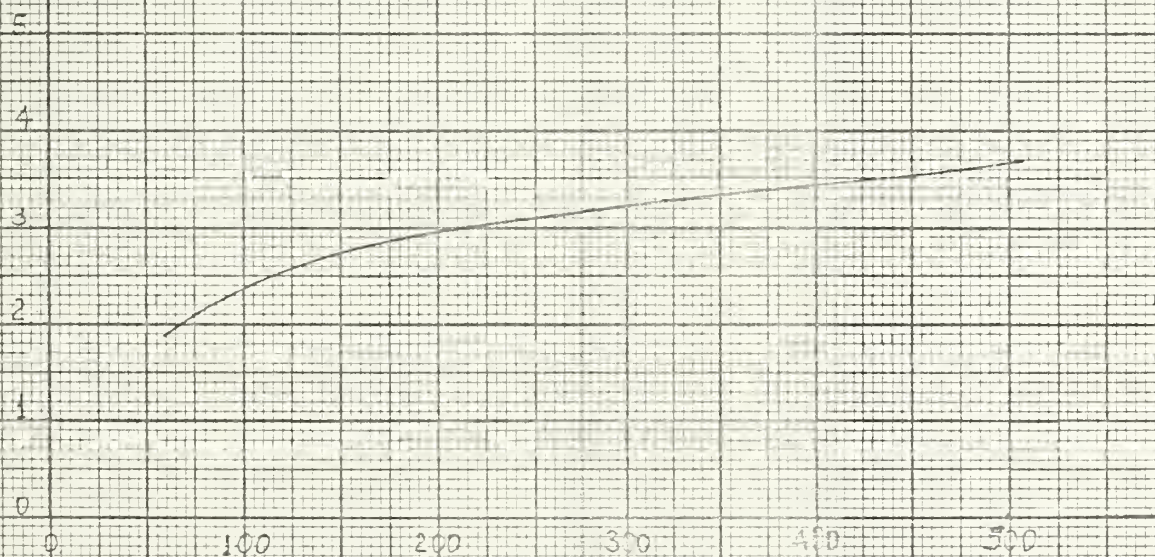


Figure (10)

-31a-

H-in	Vo-ft/sec		I'	Equation (11)or(16) Wf/H	Experimental Ref [2] Wf/H
.0643	134.08	323.05	3.806	3.682	3.542
.0644	152.60	417.16	4.326	4.253	4.120
.0638	165.06	497.30	4.723	4.684	4.650
.0638	180.06	591.78	5.152	5.123	5.166
.0647	233.00	963.55	6.574	6.458	6.420
.0635	234.08	1009.604	6.730	6.643	6.730
.0998	80.73	51.64	1.522	.771	1.046
.0985	118.95	115.08	2.272	1.515	1.890
.0984	161.70	213.10	3.092	2.690	2.755
.0983	177.90	258.46	3.405	3.053	3.330
.0982	202.60	335.90	3.881	3.564	3.760
.0984	231.13	435.38	4.420	4.096	4.300
.1728	69.69	11.79	.727	.339	.310
.1728	88.85	19.16	.927	.457	.515
.1728	130.90	41.59	1.366	.678	1.022
.1728	153.36	57.09	1.600	.780	1.257
.1728	165.70	66.64	1.730	.833	1.420
.1728	166.26	67.09	1.735	.835	1.411
.1728	171.10	71.06	1.785	.855	1.580
.1728	178.02	76.92	1.857	.883	1.715

Table (2). $b = 0.593$, $l = 5H$, $D = 300 \text{ sec}^{-1}$, $\eta = 200$.

H- in	Vo- ft/sec		Equation	Experimental,
			(11) or (16) Wf/H	Ref [2]. Wf/H
.0643	134.08	232.05	3.562	3.542
.0644	152.60	417.16	4.102	4.12
.0638	165.06	497.30	4.505	4.650
.0638	180.06	591.78	4.915	5.166
.0635	234.08	1009.60	6.318	6.730
.0998	80.73	51.64	.763	1.046
.0985	118.95	115.08	1.431	1.890
.0984	124.20	125.72	1.62	1.940
.0982	177.90	258.98	2.935	3.330
.0982	202.60	335.90	3.412	3.760
.0983	216.70	383.49	3.662	4.135
.0984	231.13	435.38	3.908	4.300
.1728	69.69	11.79	.336	.310
.1728	88.85	19.16	.452	.515
.1725	130.90	41.59	.670	1.022
.1729	165.70	66.57	.816	1.411
.1727	178.02	77.01	.864	1.715

Table (3). Theoretical vs. experimental results. $b = 0.593$, $D = 460 \text{ sec}^{-1}$,
 $l = 2.5H$, $\eta = 200$.

H- in	Vo- ft/sec		Equation (11)or(16) Wf/H	Experimental Ref [2] Wf/H
.0643	134.08	323.05	3.517	3.542
.0644	152.60	417.16	4.04	4.12
.0638	165.06	497.30	4.443	4.650
.0638	180.06	591.78	4.845	5.166
.0647	233.00	963.55	6.043	6.420
.0635	234.08	1009.60	6.220	6.730
.0998	80.73	51.64	.762	1.046
.0985	118.95	115.08	1.428	1.890
.0984	124.20	125.72	1.586	1.940
.0982	177.90	258.98	2.932	3.330
.0982	202.60	335.90	3.408	3.760
.0983	216.70	383.49	3.658	4.135
.0984	231.13	435.38	3.903	4.300
.1728	69.69	11.79	.332	.310
.1728	88.85	19.16	.443	.515
.1725	130.90	41.74	.647	1.022
.1729	165.70	66.57	.783	1.411
.1727	178.02	77.01	.828	1.715

Table (4). $b = 0.593$, $D = 460 \text{ sec}^{-1}$, $\eta = 200$

H-in	Vo-ft/sec		I'	Wf/H
.1728	69.69	11.50	.367	.6747
.1728	88.85	18.69	.486	.1077
.1728	153.36	55.69	.808	.2145
.1729	165.70	64.93	.872	.2327
.1728	166.26	65.45	.876	.2338
.1727	178.02	75.12	.938	.2508

Table (5). Theoretical results for $b = 0.25$, $D = 500 \text{ sec}^{-1}$, $l = 2.5H$,
 $\eta = 200$.

H- in	Vo- ft/sec		Equation (11)or(16) Wf/H	Experimental Ref [2] Wf/H
.0998	80.73	51.64	.765	1.046
.0985	118.95	115.08	1.459	1.890
.0984	124.20	125.72	1.618	1.940
.0982	177.90	258.98	2.976	3.330
.0983	216.70	383.49	3.718	4.135
.0982	202.60	335.90	3.462	3.760
.0984	231.13	435.38	3.969	4.300
.1728	69.69	11.79	.333	.310
.1728	88.85	19.16	.445	.515
.1725	130.90	41.74	.653	1.022
.1729	165.70	66.57	.791	1.411
.1727	178.02	77.01	.837	1.715

Table (6). $b = 0.593$, $D = 500 \text{ sec}^{-1}$, $\eta = 200$

H-in	Vo-ft/sec		I'	Wf/H
.0998	80.73	51.64	1.769	.9520
.0985	118.95	115.08	2.642	2.461
.0984	124.20	125.72	2.761	2.607
.0982	117.90	258.98	3.963	4.138
.0982	202.60	335.89	4.513	4.758
.0984	231.13	435.38	5.139	5.424
.0983	216.70	383.50	4.823	5.091

Table (7). $b = 0.75$, $D = 500 \text{ sec}^{-1}$, $l = 2.5H$, $\eta = 200$.

H-in	Vo-ft/sec		I'	Wf/H
.0998	80.73	51.64	2.074	1.931
.0985	118.95	115.08	3.097	3.605
.0984	124.20	125.72	3.237	3.786
.0982	117.90	258.98	4.646	5.773
.0982	202.60	335.89	5.290	6.617
.0984	231.13	435.38	6.023	7.540

Table (8). $b = 1.0$, $D = 500 \text{ sec}^{-1}$, $l = 2.5H$, $\eta = 200$.

D-sec ⁻¹	H-in	Vo-ft/sec	Equation (11)or(16) Wf/H	Experimental Ref [2] Wf/H
500	0.643	134.08	3.698	3.542
450	"	"	3.504	"
400	"	"	3.437	"
350	"	"	3.356	"
300	"	"	3.255	"
250	"	"	3.128	"
200	"	"	2.96	"
150	"	"	2.728	"
100	"	"	2.376	"
75	"	"	2.113	"
60	"	"	1.896	"
40	"	"	1.393	"

Table (9). Influence of D on predictions based on equations (11) or (16). $l = 2.5H$, $\eta_l = 200$.

1/H	Wf/H
1.0	3.432
2.0	3.996
3.0	4.254
4.0	4.401
5.0	4.497
6.0	4.565
7.0	4.615
8.0	4.654
9.0	4.685
10.0	4.710
11.0	4.770
12.0	4.789

Table (10). Influence of $1/H$ on predictions based on equations (11) or (16). $b = .0593$, $\tau_l = 200$, $D = 470 \text{ sec}^{-1}$. $H = .0643$, $\quad = 417.16$.

l_1 - in	H - in	Vo - ft/sec		Wf/H
0.1	.0643	134.08	323.055	3.325
0.2	"	"	"	3.698
0.3	"	"	"	3.852
0.4	"	"	"	3.935
0.5	"	"	"	3.988
0.6	"	"	"	4.025
0.8	"	"	"	4.071
1.0	"	"	"	4.100
0.1	.0644	152.60	417.16	3.807
0.2	"	"	"	4.273
0.3	"	"	"	4.469
0.5	"	"	"	4.646
0.7	"	"	"	4.728
1.0	"	"	"	4.793
0.1	.0638	180.06	591.78	4.534
0.2	"	"	"	5.150
0.3	"	"	"	5.416
0.5	"	"	"	5.662
0.1	.0638	165.06	497.30	4.173
0.2	"	"	"	4.707
0.3	"	"	"	4.935
0.5	"	"	"	5.142
0.7	"	"	"	5.239
1.0	"	"	"	5.316
0.1	.0647	233.0	963.55	5.594
0.2	"	"	"	6.499
0.3	"	"	"	6.912
0.5	"	"	"	7.303
0.7	"	"	"	7.492
1.0	"	"	"	7.644

Table (11). Influence of hinge width on the predictions based on equations (11) and (16). $b = 0.593$, $D = 500 \text{ sec}^{-1}$, $\eta = 200$.

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APPENDIX A

COMPUTER PROGRAMME FOR TABLES


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C A PROGRAMME TO COMPUTE THE PERMANENT DEFORMATION OF VISCOPLASTIC RECTAN-
C GULAR PLATE SUBJECTED TO A UNIFORMLY DISTRIBUTED DYNAMIC PRESSURE PULSE
C R=SFMI-WIDTH OF PLATE,L=SFMI-LENGTH OF PLATE,I=PLATE THICKNESS,L1=PLASTIC
C HINGE WIDTH,DVVERH=PERM. DEFLECTION OVER THICKNESS IF W.LE. 1.0,WOVERH=
C PERM. DEFLECTION OVER H IF W.GE. 1 ,VZERO= INITIAL VELOCITY,PCOL=STATIC
C COLLAPSE PRESSURE,MZERO=STATIC LIMITING MOMENT PER UNIT LENGTH OF PLATE
C MU=MASS PER UNIT AREA OF PLATE,DENS=MATERIAL DENSITY,PZERO=APPLIED PRESSUR
C E),LAMBDA=A NONDIMENSIONAL IMPULSE PARAMETER,IMPULS=A NONDIMENSIONAL PARAMET
C FR,RHO AND RHO2 ARE NONDIMENSIONAL TIMES.-
C REAL L,L1,MZERO,Iprime,IM,IMPULS,LAMDA,MU
C REAL COEFS(4),ROOTS(4),CROOTS(4),POLY(5)
C REAL GOEFS(5),FROOTS(5),GROOTS(5)
C REAL COEF(3),TAUF(3),CTAUF(3)
C DATA PFL/1.0E-27,LIMIT/50/
C READ(5,100)B,I,L1,H,DENS,VZERO,SIGMAC,P,D
C FORMAT(6F10.5,F10.4,F5.2,F5.1)
C FORMAT(/5X,'DOVERH=',F22.12)
C FORMAT(/5X,'WVERH=',F20.9)
C FORMAT(/4X,'Iprime=',F10.5)
C FORMAT(/5X,'IMPULS=',F10.5)
C FORMAT(5X,'PZERO=',F20.7)
C FORMAT(/5X,'B=',F10.5,'L=',F10.5,'L1=',F10.5,'H=',F10.5,
1/5X,'DENS=',F10.5,'VZERO=',F10.5,'SIGMAC=',F10.4,'P=',
1 F5.2,'4X,'D=',F5.1)
C FORMAT(/5X,'NEGATIVE TIME:NO THIRD STAGE')
C FORMAT(5X,'LAMBDA=',F10.5)
C FORMAT(5X,'TAUS=',E20.5)
C FORMAT(5X,'HTA=',F8.2)
C FORMAT(5X,'PCOL=',F10.3)
C FORMAT(/3X,'IERR=',I2,5X,'NROOTS=',I2)
C FORMAT('0','ALL ROOTS FOUND ARE NEGATIVE')
C FORMAT(/3X,'IERR=',I2,5X,'NROOTS=',I2)
C FORMAT(/5X,'ALL ROOTS FOUND NEGATIVE')
C WRITE(6,207) B,L,L1,H,DENS,VZERO,SIGMAC,P,D
C L=L/2.
C B=B/2.

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MARM0037
MARM0038
MARM0039
MARM0040
MARM0041
MARM0042
MARM0043
MARM0044
MARM0045
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MARM0050
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MARM0059
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MARM0065
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MARM0067
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MARM0070
MARM0071
MARM0072

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PAGE 2

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MZERO=SIGMAC*H*H/4.
BETA=B/L
TANPHI=SQRT(3+BETA*BETA)-BETA
XO=BETA*TANPHI
PHI=ATAN(TANPHI)
MU=(DENS*H)/(32.17*12.)
PCOL=((12.*MZERO)/B**2)*(1.+BETA/TANPHI)/(3.-BETA*TANPHI)
WRITE(6,217) PCOL
HTA=200.
WRITE(6,216) HTA
PZERO=HTA*PCOL
WRITE(6,206) PZERO
TAUS=(MU*MZERO*12.)/PZERO
WRITE(6,210) TAUS
LAMBDA=MU*MZERO**2*14.*L**2/(MZERO*H)
WRITE(6,209) LAMBDA
IM=TAUS*PZERO
IPRIME=IM/SQRT(MU*H*PCOL)
IMPULS=LAMBDA*(IM/(MU*MZERO*12.))**2
WRITE(6,204) IPRIME
WRITE(6,205) IMPULS
5201 C2=(2.*P/((2.*P)+1))*((H/(2.*B*L1*0))**2*(1/P))
C3=(2.*P/((2.*P)+1.))*(4/(2.*B*L1*0*TANPHI))**2*(1./P)
C1=(2.*P/((2.*P)+1.))*(H/(4.*B*L1*0*SIN(PHI)))**2*(1./P)
A1=(C2*L/R)-(C2*SIN(PHI)**2+C1+C3*COS(PHI)**2)/(2.*SIN(PHI)*COS(PHI))
1 HI)
A2=3*C2*L/B-3*C2*TANPHI+(3*C1-C2*SIN(PHI)**2-C3*COS(PHI)**2)/
1 (2.*SIN(PHI)*COS(PHI))
B1=((MU*B*B)/(12.*MZERO))*((2.-BETA*TANPHI)/(1.+BETA/TANPHI))
B2=(1.+2.*(1.-BETA*TANPHI)/(1.+BETA/TANPHI))/(3.*H*H)
B3=PZERO/PCOL
B4=(A1*BETA)/(1.+BETA/TANPHI)
B5=(A2*BETA)/(1.+BETA/TANPHI)*H*H*3.)
H1=B5/B1
H2=B4/B1
H3=B2/B1

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D1=(B3-1.)/B1
D2=(-1.)/B1
DDEF1=((D1*TAUS*TAUS)/2.)*(1.-(H2*TAUS/3.))+((H2*TAUS)**2)/12.)
1 -((H2*TAUS)**3)/60.)*+((H2*TAUS)**4)/360.)*-((D1*H3)*TAUS**4)/60.)
1 +((H3*D1*(TAUS**5)/420.)*-((H2**4)*(TAUS**5)/2520.)*+((H2*H3*D1)
1 *(TAUS**5)/126.)*-((H1*D1*D1)*(TAUS**5)/84.))
DDEF1=D1*TAUS*(1.-H2*TAUS/2.+H2**2*TAUS**2/6.-H2**3*TAUS**3/24.
1 +H2**4*TAUS**4/120.-H3*D1*TAUS**4/20.+H3*D1*TAUS**5/120.
1 -H2**4*TAUS**5/720.+H2*H3*D1*TAUS**5/36.-H1*D1*D1*TAUS**5/24.)
.PAR=H2+H1*DEFL1*DEFL1
TERM2=D2/2.-(PAR*DDEF1+H3*DDEF1**2)/2.
TERM3=(TERM2*PAR+DEFL1*DDEF1*(H3+H1*DDEF1))/(-3.)
TERM4=(2.*TERM2*DEFL1*(3.*H1*DDEF1+H3)+3.*TERM3*PAR+DDEF1**2*(H
1 1*DDEF1+H3))/(-1.)
1 AA2=2.*TERM2*TAUS
AA3=3.*TERM3*TAUS*TAUS
AA4=4.*TERM4*TAUS*TAUS*TAUS
NCOEFS=4
COEFS(1)=DDEF1
COEFS(2)=AA2
COEFS(3)=AA3
COEFS(4)=AA4
CALL ALLOC(6,6,6)
CALL PR3M(COEFS,NCOEFS,RROOTS,CROOTS,POLY,NROOTS,IERROR,REL,
1 LIMIT,DEVN)
WRITE(6,220) IERROR,NROOTS
IF(NROOTS.EQ.0) GO TO 1
IF (IERROR.NE.0) GO TO 1
CALL SORT(1,3,LL,RROOTS)
IF(LL.EQ.0) GO TO 3001
GO TO 3002
3001 WRITE(6,125)
GO TO 1
3002 CONTINUE
RROT=RROOTS(LL)
DOVERH=(DEFL1+DDEF1*TAUS*RROT+TERM2*TAUS**2*RROT**2+TERM3*TAUS**
44

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1 3*PHOT**3+TERM4*TAUS**4*RHOT**4)/H
451 WRITE(6,201) DOVERH
      IF(DOVERH .LT. 1.) GO TO 1001
45  PI1=MU*(2.-X0)/((3.-X0)*PCOL)
      PI2=2.*(3.-3.*X7+X0**2)/((3.-X0)*H)
      PI3=(H*BETA/3.)*(1/TANPHI+TANPHI)/(1+BETA/TANPHI)
      PI4=(C2/(H*(1.+BETA/TANPHI)))*(2.-2.*X0+X0/((2.*SIN(PHI))**2*(1./P)
1  *SIN(PHI)**2))
      PI5=(C2*BETA*H/(3.+3.*BETA/TANPHI))* (1./((TANPHI**2*(1.+1./P))+TANPHI)
1 HI)
      71=PI2/PI1
      72=PI3/PI1
      73=PI4/PI1
      74=PI5/PI1
      AAA2=TERM2*TAUS**2
      AAA3=TERM3*TAUS**3
      AAA4=TERM4*TAUS**4
      NCOEFS=5
      GOEFS(1)=(DEFL1-H)
      GOEFS(2)=DEFL1*TAUS
      GOEFS(3)=AAA2
      GOEFS(4)=AAA3
      GOEFS(5)=AAA4
      CALL ALLOW(7,7)
      CALL PR3M(GOEFS,NCOEFS,FROOTS,GROOTS,POLY,NROOTS,IERROR,
1 REL,LIMIT,DEVN)
      WRITE(6,227) IERROR,NROOTS
      IF (NROOTS .EQ. 0) GO TO 1
      IF (IERROR .NE. 0) GO TO 1
      CALL SORT1(1,4,LLL,FROOTS)
      IF (LLL .EQ. 0) GO TO 3011
      GO TO 3012
3011 WRITE(6,232)
      GO TO 1
3012 CONTINUE
      RHOT2=FROOTS(LLL)

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MARM0109
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 MARM0115
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DDEF L2=DDEF L1+2.*TERM2*TAUS*RHOT2+3.*TERM3*(TAUS*RHOT2)**2+
1 4.*TERM4*(TAUS*RHOT2)**3
ALPHA2=- (Z1*H+Z2/4+(Z4+Z3*H**2)*DDEF L2/H)/2.
ALPHA3=- (Z1*H+DDEF L2+Z3*H*DDEF L2**2+ALPHA2*(DDEF L2+Z4+Z3*H*H))/
1 (3.*H)
NCOEF=3
COEF(1)=DDEF L2
COEF(2)=2.*ALPHA2
COEF(3)=3.*ALPHA3
CALL ALLON (6,6,6)
CALL PRBM (COEF,NCOEF,TAUF,CTAUF,POLY,NROOTS,IEERROR,REL,LIMIT,
1 DEVN)
WRITE(6,227) IEERROR,NROOTS
IF (NROOTS .EQ. 0) GO TO 1
IF (IEERROR .NE. 0) GO TO 1
CALL SORT4 (1,2,KK,TAUF)
IF (KK .EQ. 0) GO TO 3021
GO TO 3022
3021 WRITE(6,203)
GO TO 1
3022 CONTINUE
TAUC=TAUF(KK)
555 WOVERH=1.+DDEF L2*TAUC/H+ALPHA2*TAUC**2/H+ALPHA3*TAUC**3/H
WRITE(6,203) WOVERH
1001 IF (HTA .EQ. 100.0) GO TO 225
GO TO 1
1000 CONTINUE
STOP
END

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MARM0145
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 MARM0173


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SUBROUTINE SORT1(M,N,L,A)
DIMENSION A(4)
K=0
GO TO 1
ENTRY SORT2(N)
M=1
K=1
GO TO 1
ENTRY SORT3(M,N)
K=2
NM1=N-1
DO 60 I=1,NM1
JJ=N-I
DO 50 J=1,JJ
IF (K .EQ. 2 .AND. ABS(A(J)) .GT. ABS(A(J+1))) .OR. K .NE. 2 .AND.
1 A(J) .GT. A(J+1)) GO TO 10
GO TO 50
T=A(J)
A(J)=A(J+1)
A(J+1)=T
CONTINUE
CONTINUE
IF (K .NE. 0) RETURN
ENTRY FIRSTP(N,M,L)
I=0
DO 40 I=M,N
IF (A(I) .GE. 0.) GO TO 20
CONTINUE
RETURN
L=I
RETURN
END

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SUB10001
SUB10002
SUB10003
SUB10004
SUB10005
SUB10006
SUB10007
SUB10008
SUB10009
SUB10010
SUB10011
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SUB10014
SUB10015
SUB10016
SUB10017
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SUB10029
SUB10030
SUB10031
SUB10032


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SURROUTINE SORT4(M,V,L,A)
DIMENSION A(2)
K=0
GO TO 1
ENTRY SORT5
M=1
K=1
GO TO 1
ENTRY SORT6(M,N)
K=2
NM1=N-1
DO 60 I=1,NM1
JJ=N-I
DO 50 J=1,JJ
IF (K.EQ. 2 .AND. ABS(A(J)) .GT. ABS(A(J+1))) .OR. K .NE. 2 .AND.
1 A(J) .GT. A(J+1)) GO TO 10
GO TO 50
T=A(J)
A(J)=A(J+1)
A(J+1)=T
CONTINUE
CONTINUE
IF (K .NE. 0) RETURN
ENTRY FIRST(N,M,L)
L=0
DO 40 I=M,N
IF (A(I) .GE. 0.) GO TO 20
CONTINUE
RETURN
L=I
RETURN
END

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SUB20001
SUB20002
SUB20003
SUB20004
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SUB20030
SUB20031
SUB20032

3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	470.	DATA0001
3.0	5.0625	0.15	.0644	.279	152.60	35900.0	1.0	470.0	DATA0002
3.0	5.0625	0.159	.0638	.279	165.06	35900.0	1.0	470.	DATA0003
3.0	5.0625	0.159	.0638	.279	180.06	35900.0	1.0	470.	DATA0004
3.0	5.0625	0.152	.0647	.279	233.00	35900.0	1.0	470.	DATA0005
3.0	5.0625	0.158	.0635	.279	234.08	35900.0	1.0	470.	DATA0006
3.0	5.0625	0.247	.0698	.279	80.73	33800.0	1.0	470.	DATA0007
3.0	5.0625	0.2462	.0695	.279	118.95	33800.0	1.0	470.	DATA0008
3.0	5.0625	0.246	.0694	.279	124.20	33800.0	1.0	470.	DATA0009
3.0	5.0625	0.2459	.0682	.279	177.90	33800.0	1.0	470.	DATA0010
3.0	5.0625	0.2459	.0692	.279	202.60	33800.0	1.0	470.	DATA0011
3.0	5.0625	0.246	.0683	.279	216.70	33800.0	1.0	470.0	DATA0012
3.0	5.0625	0.2461	.0684	.279	231.13	33900.0	1.0	470.0	DATA0013
3.0	5.0625	0.432	.1728	.279	69.69	36800.0	1.0	470.0	DATA0014
3.0	5.0625	0.432	.1728	.279	88.85	36800.0	1.0	470.	DATA0015
3.0	5.0625	0.43	.06725	.279	130.90	36800.0	1.0	470.	DATA0016
3.0	5.0625	0.4321	.06729	.279	165.70	36800.0	1.0	470.	DATA0017
3.0	5.0625	0.4319	.06727	.279	178.02	36800.0	1.0	470.	DATA0018
3.0	5.0625	0.25	.06998	.279	80.73	33800.0	1.0	460.	DATA0019
3.0	5.0625	0.25	.06985	.279	118.95	33800.0	1.0	460.	DATA0020
3.0	5.0625	0.23	.06984	.279	124.20	33800.0	1.0	460.	DATA0021
3.0	5.0625	0.25	.06982	.279	177.90	33800.0	1.0	460.0	DATA0022
3.0	5.0625	0.25	.06982	.279	202.60	33800.0	1.0	460.	DATA0023
3.0	5.0625	0.25	.06983	.279	216.70	33800.0	1.0	460.0	DATA0024
3.0	5.0625	0.25	.06984	.279	231.13	33800.0	1.0	460.	DATA0025
3.0	5.0625	0.3	.1728	.279	69.69	36800.0	1.0	460.0	DATA0026
3.0	5.0625	0.3	.1728	.279	88.85	36800.0	1.0	460.	DATA0027
3.0	5.0625	0.3	.06725	.279	130.90	36800.0	1.0	460.	DATA0028
3.0	5.0625	0.30	.06729	.279	165.70	36800.0	1.0	460.	DATA0029
3.0	5.0625	0.30	.06727	.279	178.02	36800.0	1.0	460.	DATA0030
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	460.	DATA0031
3.0	5.0625	0.15	.0644	.279	152.60	35900.0	1.0	460.	DATA0032
3.0	5.0625	0.15	.0638	.279	165.06	35900.0	1.0	460.	DATA0033
3.0	5.0625	0.15	.0638	.279	180.06	35900.0	1.0	460.	DATA0034
3.0	5.0625	0.15	.0647	.279	233.00	35900.0	1.0	460.	DATA0035
3.0	5.0625	0.15	.0635	.279	234.08	35900.0	1.0	460.	DATA0036

3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	400.	DATA0037
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	350.	DATA0038
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	300.0	DATA0039
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	250.	DATA0040
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	200.	DATA0041
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	150.	DATA0042
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	100.	DATA0043
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	75.	DATA0044
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	60.	DATA0045
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	40.2	DATA0046
3.0	5.0625	0.15	.0643	.279	134.08	35900.0	1.0	500.	DATA0047
1.250	5.	0.3	.1728	.279	69.69	36800.0	1.0	500.	DATA0048
1.250	5.	0.3	.1728	.279	88.85	36800.0	1.0	500.	DATA0049
1.250	5.	0.3	.1728	.279	153.36	36800.0	1.0	500.	DATA0050
1.250	5.	0.3	.1729	.279	145.70	36800.0	1.0	500.	DATA0051
1.250	5.	0.3	.1728	.279	146.26	36800.0	1.0	500.	DATA0052
1.250	5.	0.3	.1727	.279	178.02	36800.0	1.0	500.	DATA0053
0.625	2.511	0.25	.106	.279	391.6	36400.	1.0	500.	DATA0054
0.625	2.511	0.25	.106	.279	257.75	36400.	1.0	500.	DATA0055
3.7970	5.0625	0.25	0.0998	.279	80.73	33800.0	1.0	500.	DATA0056
3.7970	5.0625	0.25	0.0985	.279	118.95	33800.0	1.0	500.	DATA0057
3.7970	5.0625	0.23	0.0984	.279	124.20	33800.0	1.0	500.	DATA0058
3.7970	5.0625	0.25	0.0982	.279	177.90	33800.0	1.0	500.	DATA0059
3.797	5.0625	0.25	0.0982	.279	202.60	33800.0	1.0	500.	DATA0060
3.797	5.0625	0.25	0.0984	.279	231.13	33800.0	1.0	500.	DATA0061
3.7970	5.0625	0.25	.0993	.279	216.70	33800.0	1.0	500.0	DATA0062
5.0625	5.0625	0.25	0.0998	.279	90.73	33800.0	1.0	500.	DATA0063
5.0625	5.0625	0.25	0.0985	.279	118.95	33800.0	1.0	500.	DATA0064
5.0625	5.0625	0.23	0.0984	.279	124.20	33800.0	1.0	500.	DATA0065
5.0625	5.0625	0.25	0.0982	.279	177.90	33800.0	1.0	500.	DATA0066
5.0625	5.0625	0.25	0.0982	.279	202.60	33800.0	1.0	500.	DATA0067
5.0625	5.0625	0.25	0.0984	.279	231.13	33800.0	1.0	500.	DATA0068
5.0625	5.0625	0.3	.1728	.279	166.26	36800.0	1.0	500.0	DATA0069
5.0625	5.0625	0.3	.1728	.279	88.85	36800.0	1.0	500.	DATA0070
5.0625	5.0625	0.3	.1728	.279	69.69	36800.0	1.0	500.0	DATA0071
5.0625	5.0625	0.3	.1731	.279	171.16	36800.0	1.0	500.0	DATA0072
5.0625	5.0625	0.3	.1727	.279	178.02	36800.0	1.0	500.0	DATA0073

3.0	60.00	0.25	0.0998	.279	90.73	33800.0	1.0	40.	DATA0073
3.	60.00	0.25	0.0995	.279	118.95	33800.0	1.0	40.	DATA0074
3.000	60.0000	0.23	0.0984	.279	124.20	33800.0	1.0	40.0	DATA0075
3.00	60.0000	0.25	0.0982	.279	177.90	33800.0	1.0	40.	DATA0076
3.00	60.0000	0.25	0.0982	.279	202.60	33800.0	1.0	40.	DATA0077
3.00	60.0000	0.25	0.0984	.279	231.13	33800.0	1.0	40.	DATA0078
3.0	5.0625	0.1	.0643	.279	134.08	35900.0	1.0	500.	DATA0079
3.0	5.0625	0.2	.0643	.279	134.08	35900.0	1.0	500.	DATA0080
3.0	5.0625	0.3	.0643	.279	134.08	35900.0	1.0	500.	DATA0081
3.0	5.0625	0.5	.0643	.279	134.08	35900.0	1.0	500.	DATA0082
3.0	5.0625	0.7	.0643	.279	134.08	35900.0	1.0	500.	DATA0083
3.0	5.0625	1.0	.0643	.279	134.08	35900.0	1.0	500.	DATA0084
3.0	5.0625	0.1	.0644	.279	152.60	35900.0	1.0	500.	DATA0085
3.0	5.0625	0.2	.0644	.279	152.60	35900.0	1.0	500.	DATA0086
3.0	5.0625	0.3	.0644	.279	152.60	35900.0	1.0	500.	DATA0087
3.0	5.0625	0.5	.0644	.279	152.60	35900.0	1.0	500.	DATA0088
3.0	5.0625	0.7	.0644	.279	152.60	35900.0	1.0	500.	DATA0089
3.0	5.0625	1.0	.0644	.279	152.60	35900.0	1.0	500.	DATA0090
3.0	5.0625	0.1	.0638	.279	180.06	35900.0	1.0	500.	DATA0091
3.0	5.0625	0.2	.0638	.279	180.06	35900.0	1.0	500.	DATA0092
3.0	5.0625	0.3	.0638	.279	180.06	35900.0	1.0	500.	DATA0093
3.0	5.0625	0.5	.0638	.279	180.06	35900.0	1.0	500.	DATA0094
3.0	5.0625	0.7	.0638	.279	180.06	35900.0	1.0	500.	DATA0095
3.0	5.0625	1.0	.0638	.279	180.06	35900.0	1.0	500.	DATA0096
3.0	5.0625	0.1	.0638	.279	165.06	35900.0	1.0	500.	DATA0097
3.0	5.0625	0.2	.0638	.279	165.06	35900.0	1.0	500.	DATA0098
3.0	5.0625	0.3	.0638	.279	165.06	35900.0	1.0	500.	DATA0099
3.0	5.0625	0.5	.0638	.279	165.06	35900.0	1.0	500.	DATA0100
3.0	5.0625	0.7	.0638	.279	165.06	35900.0	1.0	500.	DATA0101
3.0	5.0625	1.0	.0638	.279	165.06	35900.0	1.0	500.	DATA0102
3.0	5.0625	0.1	.0647	.279	233.00	35900.0	1.0	500.	DATA0103
3.0	5.0625	0.2	.0647	.279	233.00	35900.0	1.0	500.	DATA0104
3.0	5.0625	0.3	.0647	.279	233.00	35900.0	1.0	500.	DATA0105
3.0	5.0625	0.5	.0647	.279	233.00	35900.0	1.0	500.	DATA0106
3.0	5.0625	0.7	.0647	.279	233.00	35900.0	1.0	500.	DATA0107
3.0	5.0625	1.0	.0647	.279	233.00	35900.0	1.0	500.	DATA0108

3.0	5.0625	0.1	0.0635	.279	234.08	35900.0	1.0	500.	DATA0109
3.0	5.0625	0.2	0.0635	.279	234.08	35900.0	1.0	500.	DATA0110
3.0	5.0625	0.3	0.0635	.279	234.08	35900.0	1.0	500.	DATA0111
3.0	5.0625	0.5	0.0635	.279	234.08	35900.0	1.0	500.	DATA0112
3.0	5.0625	0.7	0.0635	.279	234.08	35900.0	1.0	500.	DATA0113
3.0	5.0625	1.0	0.0635	.279	234.08	35900.0	1.0	500.	DATA0114
3.0	5.0625	0.25	0.0998	.279	80.73	33800.0	1.0	500.0	DATA0115
3.0	5.0625	0.25	0.0995	.279	118.95	33800.0	1.0	500.	DATA0116
3.0	5.0625	0.23	0.0984	.279	124.20	33800.0	1.0	500.	DATA0117
3.0	5.0625	0.25	0.0982	.279	177.90	33800.0	1.0	500.	DATA0118
3.0	5.0625	0.25	0.0982	.279	202.60	33800.0	1.0	500.	DATA0119
3.0	5.0625	0.25	.0983	.279	216.70	33800.0	1.0	500.0	DATA0120
3.0	5.0625	0.25	0.0984	.279	231.13	33800.0	1.0	500.	DATA0121
3.0	5.0625	0.3	.1728	.279	69.69	36800.0	1.0	500.0	DATA0122
3.0	5.0625	0.3	.1728	.279	88.85	36800.0	1.0	500.	DATA0123
3.0	5.0625	0.3	0.1725	.279	130.90	36800.0	1.0	500.	DATA0124
3.0	5.0625	0.30	0.1729	.279	165.70	36800.0	1.0	500.	DATA0125
3.0	5.0625	0.30	0.1727	.279	178.02	36800.0	1.0	500.	DATA0126
									DATA0127

APPENDIX B

COMPUTER PROGRAMME FOR GRAPHS


```

C A PROGRAMME FOR PLOTTING DEFLECTION-THICKNESS RATIO VS. IPRIME
C FOR DIFFERENT VALUES OF HTA (SEE PREVIOUS PROGRAMME MARM)
C R=SEMI-WIDTH OF PLATE,L=SEMI-LENGTH OF PLATE,H=PLATE THICKNESS,L1=PLASTIC
C HINGE WIDTH,COVERH=PERM. DEFLECTION OVER THICKNESS IF W.LE. 1.0,WVERH=
C PERM. DEFLECTION OVER H IF W.GE. 1. VZERO= INITIAL VELOCITY,PCOL=STATIC
C COLLAPSE PRESSURE,VZERO=STATIC LIMITING MOMENT PER UNIT LENGTH OF PLATE
C MU=MASS PER UNIT AREA OF PLATE,DENS=MATERIAL DENSITY,PZERO=APPLIED PRESSUR
C E),LAMBDA=A NONDIMENSIONAL IMPULSE PARAMETER,IMPULS=A NONDIMENSIONAL PARAMET
C ER,PHI AND PHI2 ARE NONDIMENSIONAL TIMES.-
C CALLZ L,L1,MZERO,IPRIME,IMPULS,LAMBDA,MU
C DIMENSION COEF(4),RGPTS(4),CROPTS(4),POLY(4),
C 1 GTFES(5),CROPTS(5),POL(5),CROPTS(5),COEF(3),TAUF(3),CTAUF(3),
C 1 POLY(3)
C DIMENSION YVER(4,10),XOP(10),TIT1(0),TIT2(0),TIT3(0),HTA(4),
C 1 YV(10)
C DATA TIT1/ ASPECT RATIO=.503
C DATA TIT2/ IPRIME
C DATA TIT3/ DEFLECTION OVER THICKNESS
C 7000 I=1,4
C YV2(I,1)=0.
C CALL STOPV('M9706-5246',9,3)
C HTA(1)=20.
C HTA(2)=100.
C HTA(3)=200.
C HTA(4)=300.
C XOP(1)=0.
C 1000 ISTAV=2.10
C PZERO(5,100)P,L,L1,H,DENS,VZERO,SIGMAC,P,D
C FORMAT(6F10.5,F10.4,F5.2,F5.1)
C 201 FORMAT(//5X,WVERH=,F22.12)
C 203 FORMAT(//5X,WVERH=,F20.9)
C 205 FORMAT(//4X,IPRIME=,F10.5)
C 207 FORMAT(//5X,IMPULS=,F10.5)
C 209 FORMAT(//5X,PZERO=,F20.7)
C 211 FORMAT(//5X,P=,F10.5,5X,L=,F10.5,5X,L1=,F10.5,5X,H=,F10.5,
C 1/5X,DENS=,F10.5,5X,VZERO=,F10.5,5X,SIGMAC=,F10.4,5X,P=,

```



```

r 201 C2=(2.*P/(2.*P)+1))*((H/(2.*9*L1*P)))*(1/P))
C3=(2.*P/(2.*P)+1.))*((H/(2.*3*L1*P*TANPHI)))*(1./P)
C1=(2.*P/(2.*P)+1.))*((H/(4.*9*L1*P*SIN(PHI)))*(1./P)
A1=(C2*L/3)-(C2*SIN(PHI))*2+C1+C3*COS(PHI))*2/(2.*SIN(PHI)*COS(PHI))
1 HI))
A2=2*(C2*L/P-3*C2*TANPHI+(3*L1-C2*SIN(PHI))*2-C3*COS(PHI))*2/
1 (2.*SIN(PHI)*COS(PHI))
P1=((4*H*P*P)/(12.*P*P*P))*((2.-P*TAN(PHI))/(1.+3*TA/TANPHI))
P2=(1.+2.*(1.-P*TA/TANPHI))/(1.+P*TA/TANPHI))/(3.*H*H)
P3=P*P*P/P*P*P
P4=(A1+P*P*TA)/(1.+P*TA/TANPHI)
P5=(A2+P*P*TA)/(1.+P*TA/TANPHI)*H*H*3.)
H1=P5/B1
H2=P4/P1
H3=P2/P1
D1=(P3-1.)/R1
D2=(-1.)/R1
DDEF1=((D1*TAUS*TAUS)/2.)*(1.-(P2*TAUS/3.)+(H2*TAUS)*2/12.)
1 -(H2*TAUS)*3/60.)+(H2*TAUS)*2/360.)-(D1*H3)*TAUS**4/60.)
1 +(H3*D1)*(TAUS**5)/620.)-(H2**4)*(TAUS**5)/2520.)+(H2*H3*D1)
1 *(TAUS**5)/126.)-(H1*D1)*(TAUS**5)/P4.)
DDEF11=D1*TAUS*(1.-H2*TAUS/2.+H2**2*TAUS**2/6.-H2**3*TAUS**3/24.
1 +H2**4*TAUS**4/120.-P3*D1*TAUS**4/20.+H3*D1*TAUS**5/120.
1 -H2**4*TAUS**5/720.+H2*H3*D1*TAUS**5/36.-H1*D1*TAUS**5/24.)
DAP=H2+H1*DDEF11*DDEF11
TERM2=D2/2.-(PAP*DDEF11+H3*DDEF11**2)/2.
TERM3=(TERM2*PAP+DDEF11*DDEF11*(H3+H1*DDEF11))/(-3.)
TERM4=(2.*TERM2*DDEF11*(3.*H1*DDEF11+H3)+3.*TERM3*PAP+DDEF11**2*(H
1 1*DDEF11+H3))/(-1.)
AA2=2.*TERM2*TAUS
AA3=3.*TERM3*TAUS*TAUS
AA4=2.*TERM4*TAUS*TAUS*TAUS
NCOFFS=4
COFFS(1)=DDEF11
COFFS(2)=AA2
COFFS(3)=AA3

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PAGE 3


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COEF5(4)=AA4
CALL OPBM(COEF5,NCOEF5,PROOTS,ROOTS,POLY,NROOTS,IEPERR)
WRITE(6,220) IEPERR,NROOTS
IF(NROOTS.EQ. 0) GO TO 1
IF(IEPERR.NE. 0) GO TO 1
CALL SUBT(1,3,LI,ROOTS)
IF(LI.EQ. 0) GO TO 3001
GO TO 3002
3001 WRITE(6,125)
GO TO 1
3002 CONTINUE
PHCT=PROOTS(1,1)
DOVPRM=(DEFL1+DEFL1*TAUS*RHOT+TERM2*TAUS**2*PHCT**2+TERM3*TAUS**
1 3*RHOT**3+TERM4*TAUS**4*PHCT**4)/4
WRITE(6,201) DOVPRM
IF(DOVPRM.LT. 1.) GO TO 1001
PI1=MIY*(2.-X0)/(3.-X0)*PCOI
PI2=2.*(3.-3.*X1+X0**2)/(3.-X0)*H
PI3=(H*BETA/3.)*(1/TANPHI+TANPHI)/(1+BETA/TANPHI)
PI4=(C2/(H*(1.+BETA/TANPHI)))*(2.-2.*X0+X0/(2.*SIN(PHI)))*(1./P)
1  +SIN(PHI)**2)
PI5=(C2*BETA/H/(3.+3.*BETA/TANPHI))* (1./(TANPHI**2*(1.+1./P))+TANPHI)
1  HI)
71=PI2/PI1
72=PI3/PI1
73=PI4/PI1
74=PI5/PI1
AA2=TERM2*TAUS**2
AA3=TERM3*TAUS**3
AA4=TERM4*TAUS**4
NCOEF5=7
COEF5(1)=(DEFL1-H)
COEF5(2)=DEFL1*TAUS
COEF5(3)=AA2
COEF5(4)=AA3
COEF5(5)=AA4

```



```

CALL ORPM(GOEF,NGOEF,FRQNTS,GROUPTS,POL,NRQNTS,ERROR)
WRITE(6,227) IFERRP,NRQNTS
IF (NRQNTS.EQ.0) GO TO 1
IF (IFERRP.NE.0) GO TO 1
CALL SORT1(1,4,LL,FRQNTS)
IF (LL.EQ.0) GO TO 3011
GO TO 3012
3011 WRITE(6,232)
GO TO 1
3012 CONTINUE
CHOT2=FRQNTS(LL)
DDEFEL2=DDEFEL1+2.*TERM2*TAUS*RHOT2+3.*TERM43*(TAUS*RHOT2)**2+
1 4.*TERM44*(TAUS*RHOT2)**3
ALPHA2=- (71*H+72/H+(74+73*H**2)*DDEFEL2/H)/2.
ALPHA3=- (71*H+DDEFEL2+73*H*DDEFEL2**2+ALPHA2*(DDEFEL2+74+73*H**2))/
1 (3.*H)
NGOEF=3
COEF(1)=DDEFEL2
COEF(2)=2.*ALPHA2
COEF(3)=3.*ALPHA3
CALL PRPH(COEF,NGOEF,TAUF,CTAUF,FOLY,NRQNTS,ERROR)
WRITE(6,227) IFERRP,NRQNTS
IF (NRQNTS.EQ.0) GO TO 1
IF (IFERRP.NE.0) GO TO 1
CALL SORT4(1,3,KK,TAUF)
IF (KK.EQ.0) GO TO 3021
GO TO 3022
3021 WRITE(6,20P)
GO TO 1
3022 CONTINUE
TAUC=TAUF(KK)
WCOVERH=1.+DDEFEL2*TAUC/H+ALPHA2*TAUC**2/H+ALPHA3*TAUC**3/H
WRITE(6,203) WCOVERH
YVERP(JSTAV,ISTAV)=WCOVERH
GO TO 1002
1001 YVERP(JSTAV,ISTAV)=DCOVERH

```



```

1002 CONTINUE
1000 CONTINUE
WRITE(6,240) (XOP(I),I=1,18)
CALL MAX(XOP,18,XMIN,XMAX)
CALL MAX(YVER,72,YMIN,YMAX)
DO 1003 I=1,4
DC 1004 J=1,18
YV(J)=YVER(I,J)
WRITE(6,241)(YV(I),I=1,18)
1003 CALL DIR2(1,1,1,0,13,0,XMAX,0,YMAX,1,1,0,0,-1,-1,4,4,XOR,
1 YV,TIT1,TIT2,TIT3,36,36,36)
CALL PLTND(NFI)
STOP
END

```

```

MAIN0181
MAIN0182
MAIN0183
MAIN0184
MAIN0185
MAIN0186
MAIN0187
MAIN0188
MAIN0189
MAIN0190
MAIN0191
MAIN0192
MAIN0193
MAIN0194

```



```

SUBROUTINE MAX (A,NA,AL,AT)
DIMENSION A(1)
WRITE (*,4006)
4006 FORMAT ('ENTER MAX')
AMAX=0.
DO 10 I=1,NA
P=ABS(A(I))
IF(P-AMAX) 10,10,11
11 AMAX=P
10 CONTINUE
R=1.
J=-1
IF(AMAX-P) 12,13,13
13 R=P/10.
J=J+1
IF(AMAX-P) 14,13,13
14 C=P/10.
I=AMAX+C
IF(J) 20,20,21
21 DO 22 K=1,J
22 I=I/10
OR 22 K=1,J
23 I=I*10
GO TO 20
12 R=P/10.
J=J+1
IF(AMAX-P) 12,15,15
15 K=J+1
AT=AMAX+B
DO 24 L=1,K
24 AT=AT*10.0
I=AT
AT=I
R=25 L=1,K
25 AT=AT/10
AL=-AT

```

```

MAX10001
MAX10002
MAX10003
MAX10004
MAX10005
MAX10006
MAX10007
MAX10008
MAX10009
MAX10010
MAX10011
MAX10012
MAX10013
MAX10014
MAX10015
MAX10016
MAX10017
MAX10018
MAX10019
MAX10020
MAX10021
MAX10022
MAX10023
MAX10024
MAX10025
MAX10026
MAX10027
MAX10028
MAX10029
MAX10030
MAX10031
MAX10032
MAX10033
MAX10034
MAX10035
MAX10036

```



```

AT=AI
GO TO 30
20 AL=-I
AT=I
30 RETURN
END

```

```

MAX10037
MAX10038
MAX10039
MAX10040
MAX10041
MAX10042

```



```

      SUBROUTINE DIR2  (NFC,IND,NGRAPH,NGO,NPOINT,XL,XR,YB,YT,EX,OY,
      * N,M,I,J,NX,NY,X,Y,TIT,TITX,TITY,NT,NTX,NTY)
      DIMENSION X(1),Y(1),TIT(1),TITX(1),TITY(1)
      WRITE (6,6007)
      COMMON /POINT/ OIDO
      INDA=0
      GO TO (1,2,3,4), IND
      1 CALL SMXYV(0,0)
      GO TO 5
      2 CALL SMXYV(0,1)
      GO TO 5
      3 CALL SMXYV(1,0)
      GO TO 5
      4 CALL SMXYV(1,1)
      5 CONTINUE
      CALL SETMTV(150,100,150,150)
      IF(NFC-1) 10,10,20
      10 NFA=2
      GO TO 30
      20 NFA=4
      30 CALL GRIDIV(NFA,XL,XR,YB,YT,OX,OY,N,M,I,J,NX,NY)
      CALL RITE2V(300,125,1000,0,2,TITX,1,TITX,NLAST)
      CALL RITE2V(125,250,1000,50,2,ITY,1,TITY,NLAST)
      CALL RITE2V(250,925,1000,0,2,NT,1,TIT,NLAST)
      CALL INCRV(P,4)
      NAD=NGRAPH+NGO
      GO 7 IT=1,NAL
      NAUX=NPOINT-1
      DO 9 K=1,NAUX
      IAUX=(IT-1)*NPOINT+K
      XI=X(K)
      X2=X(K+1)
      Y1=Y(ITAUX)
      Y2=Y(ITAUX+1)
      IE=(Y1-YT) 100,100,101
      IC=(Y2-YT) 110,110,102
      100

```

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103 X2=(X2-X1)*(YT-Y1)/(Y2-Y1)+X1
    Y2=YT
    GO TO 110
101 IF(Y2-YT) 104,104,105
104 X1=(X2-X1)*(YT-Y1)/(Y2-Y1)+X1
    Y1=YT
    GO TO 110
105 IND=1
110 CONTINUE
    IF(Y1-Y3) 200,201,201
200 IF(Y2-Y3) 205,203,203
205 IND=1
    GO TO 210
203 X1=(X2-X1)*(Y3-Y1)/(Y2-Y1)+X1
    Y1=Y3
    GO TO 210
201 IF(Y2-Y3) 204,210,210
204 X2=(X2-X1)*(Y3-Y1)/(Y2-Y1)+X1
    Y2=Y3
210 CONTINUE
    IF(IND) 303,303,302
303 IF(1-NSP4) 300,300,301
300 CALL LINEV(NXV(X1),NYV(Y1),IXV(X2),NYV(Y2))
    CALL LINEV(IXV(X1),NYV(Y1),NXV(X2),NYV(Y2))
    GO TO 302
301 CALL DATLIN(IXV(X1),NYV(Y1),NXV(X2),NYV(Y2))
    CALL DATLIN(NXV(X1),NYV(Y1),IXV(X2),NYV(Y2))
302 IND=0
8 CONTINUE
7 CONTINUE
    STOP
END

```

DIR20037
 DIR20038
 DIR20039
 DIR20040
 DIR20041
 DIR20042
 DIR20043
 DIR20044
 DIR20045
 DIR20046
 DIR20047
 DIR20048
 DIR20049
 DIR20050
 DIR20051
 DIR20052
 DIR20053
 DIR20054
 DIR20055
 DIR20056
 DIR20057
 DIR20058
 DIR20059
 DIR20060
 DIR20061
 DIR20062
 DIR20063
 DIR20064
 DIR20065
 DIR20066
 DIR20067
 DIR20068


```

SUBROUTINE SORT1(M,N,L,A)
DIMENSION A(4)
K=0
GO TO 1
ENTRY SORT2(N)
M=1
K=1
GO TO 1
ENTRY SORT2(M,N)
K=2
NM1=N-1
DO 60 I=1,NM1
JJ=N-I
DO 50 J=1,JJ
IF (K.EQ. 2 .AND. ABS(A(J)) .GT. ABS(A(J+1))) .OR. K.NE. 2 .AND.
1 A(J) .GT. A(J+1)) GO TO 10
GO TO 50
T=A(J)
A(J)=A(J+1)
A(J+1)=T
CONTINUE
CONTINUE
IF (K.NE. 0) RETURN
ENTRY FIRSTP(M,M,L)
L=0
DO 40 I=M,N
IF (A(I) .GE. 0.) GO TO 20
CONTINUE
RETURN
L=1
RETURN
END

```

SOR10001
 SOR10002
 SOR10003
 SOR10004
 SOR10005
 SOR10006
 SOR10007
 SOR10008
 SOR10009
 SOR10010
 SOR10011
 SOR10012
 SOR10013
 SOR10014
 SOR10015
 SOR10016
 SOR10017
 SOR10018
 SOR10019
 SOR10020
 SOR10021
 SOR10022
 SOR10023
 SOR10024
 SOR10025
 SOR10026
 SOR10027
 SOR10028
 SOR10029
 SOR10030
 SOR10031
 SOR10032


```

SUBROUTINE SORT4(M,N,L,A)
DIMENSION A(2)
K=0
GO TO 1
ENTRY SORT5
M=1
K=1
GO TO 1
ENTRY SORT6(M,N)
K=2
1  N*1=N-1
   DO 40 I=1,N*1
      JJ=N-I
      DO 50 J=1,JJ
         IF (K .EQ. 2 .AND. ABS(A(J)) .GT. ABS(A(J+1))) .CR. K .NE. 2 .AND.
1  A(J) .GT. A(J+1)) GO TO 10
         GO TO 50
      T=A(J)
      A(J)=A(J+1)
      A(J+1)=T
      CONTINUE
      CONTINUE
      IF (K .NE. 0) RETURN
      ENTRY FIRST(N,M,L)
      L=0
      DO 40 I=M,N
         IF (A(I) .GE. 0.) GO TO 20
      CONTINUE
      RETURN
20  L=L+1
      RETURN
      END

```

SORT40001
 SORT40002
 SORT40003
 SORT40004
 SORT40005
 SORT40006
 SORT40007
 SORT40008
 SORT40009
 SORT40010
 SORT40011
 SORT40012
 SORT40013
 SORT40014
 SORT40015
 SORT40016
 SORT40017
 SORT40018
 SORT40019
 SORT40020
 SORT40021
 SORT40022
 SORT40023
 SORT40024
 SORT40025
 SORT40026
 SORT40027
 SORT40028
 SORT40029
 SORT40030
 SORT40031
 SORT40032

3.0	5.0625	0.158	0.0635	.270	76.08	35500.0	1.0	470.	DATA0001
3.0	5.0625	0.16	.0643	.270	98.00	35500.0	1.0	470.	DATA0002
3.0	5.0625	0.16	.0643	.270	120.00	35500.0	1.0	470.	DATA0003
3.0	5.0625	0.16	.0643	.270	134.08	35500.0	1.0	470.	DATA0004
3.0	5.0625	0.16	.0644	.270	142.00	35500.0	1.0	470.0	DATA0005
3.0	5.0625	0.16	.0644	.270	152.40	35500.0	1.0	470.0	DATA0006
3.0	5.0625	0.16	.0644	.270	160.00	35500.0	1.0	470.0	DATA0007
3.0	5.0625	0.159	.0639	.270	145.06	35500.0	1.0	470.	DATA0008
3.0	5.0625	0.159	.0639	.270	170.00	35500.0	1.0	470.	DATA0009
3.0	5.0625	0.159	.0638	.270	180.06	35500.0	1.0	470.	DATA0010
3.0	5.0625	0.159	.0638	.270	197.06	35500.0	1.0	470.	DATA0011
3.0	5.0625	0.159	.0639	.270	209.06	35500.0	1.0	470.	DATA0012
3.0	5.0625	0.162	.0647	.270	233.00	35500.0	1.0	470.	DATA0013
3.0	5.0625	0.159	0.0635	.270	245.08	35500.0	1.0	470.	DATA0014
3.0	5.0625	0.162	.0647	.270	259.00	35500.0	1.0	470.	DATA0015
3.0	5.0625	0.142	.0647	.270	270.00	35500.0	1.0	470.	DATA0016
3.0	5.0625	0.153	0.0635	.270	282.08	35500.0	1.0	470.	DATA0017

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DISPLAY

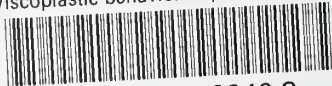
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